

F.E.D. Vignette #4:

The Gödelian Dialectic ***of*** ***the Standard Arithmetics.***

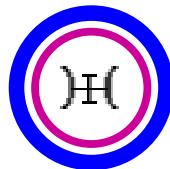
A *Dialectical* 'Meta-Model' of the F.E.D. 'Meta-Systematic *Dialectical Method of Presentation*' of the *Axioms-Systems Progression* of the Standard Arithmetics, Using F.E.D.'s '*First Dialectical Algebra*'.

'Habilitation-«Fest»' Essay

by **Miguel Detonacciones**

of

Foundation *Encyclopedia Dialectica*



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A *Dialectical* ‘*Meta-Model*’ of the **F.E.D.** ‘*Meta-Systematic Dialectical Method of Presentation*’ of the *Axioms-Systems Progression* of the Standard Arithmetics, Using **F.E.D.**’s ‘*First Dialectical Algebra*’.

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About the Author. Miguel Detonacciones has been a longtime frequent visitor to the www.dialectics.org website, and to the www.adventures-in-dialectics.org website, and a commentator upon -- and, from outside of the **F.E.D.** research collective, a creative contributor to -- the fruits of the **F.E.D.** project. That project, 'The Seldon Project', is the already six decades deep, and continuing, research effort, to which Miguel has already contributed, while he was still a non-member of **F.E.D.**, by his many posts to many internet forums -- including to some that he founded -- by his blogging, by his correspondence, and by related activities.

In July of the year **2012 C.E./B.U.E.**, Miguel Detonacciones was inducted into full membership in the **Foundation**, by unanimous acclaim of the **Foundation** then-membership.

The present text is Miguel's instance of the traditional "'habilitation'" contribution to 'The Seldon Project', which new members prepare as the fruition of their first **Foundation** task after joining **F.E.D.**. Its content was presented by Miguel Detonacciones, in an oral/visual summary format, to the rest of the **F.E.D.** membership, during our August «*fête*», our customary feast, held in honor of the union of each new member of **F.E.D.** with the **Foundation** community. It was thereafter formally adopted, as **Foundation Encyclopedia Dialectica Vignette #4**, by unanimous vote of the **F.E.D.** General Council, and posted in PDF-file format to **F.E.D.**-affiliated sites across the world wide web.

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Author's Preface. It is my privilege, and my high honor, to be the first author commissioned by the **F.E.D.** General Council to write for **F.E.D.**'s public upon the topic of this **F.E.D. Vignette**, and to be the first to publish writing on this topic via this -- now venerable -- website, a website which has, for the last ~8 years, archived, for that public, the published portions of the **F.E.D. «opus»**. It is my honor as well to do so to mark my unanimous election, by the existing membership of the **F.E.D.** collective, for induction into full membership in the 'meta-Pythagorean brotherhood-sisterhood'/'meta-Platonian Academy' that is **F.E.D.!**

The topic of this essay is one that is central to the **F.E.D. «opus»**, but it is also one upon which **F.E.D.** has not previously made public its principal findings.

And this, despite the fact that the entire second volume of **F.E.D.**'s forthcoming treatise -- **Dialectical Ideography: A Contribution to the Immanent Critique of Arithmetic** [DIACTTICOA] -- per that second volume's specific title, **The Meta-Evolution of Arithmetics**, pivots upon an exposition of this topic. This, despite the fact, too, that the exposition of this topic is slated for a more pictorial presentation in the final chapter of volume 2 -- entitled **Dialectical Meta-Models of the Human Phenome** -- of that other multi-volume work by **F.E.D.**, that work of '**Dialectical Pictography**' whose overall title is **A Dialectical "Theory of Everything" -- Meta-Genealogies of the Universe and of Its Sub-Universes: A Graphical Manifesto** [ADTOE-MOTUAOISU-AGM], of which the initial volume is already extant [although out of print, at present]. Please see --

Table of Contents, DIACTTICOA: <http://www.adventures-in-dialectics.org/Adventures-In-Dialectics/DiaRith/PrescriptsOverview/PrescriptsOverview.htm>
 Table of Contents, ADTOE-MOTUAOISU-AGM, vol. 2

I have been privileged to have been allowed to review, even before my induction into membership in the **Foundation**, as yet unpublished manuscripts for that volume, and for that chapter, which prepared me for the writing of the present essay. What is presented herein neither pre-empts, nor reproduces in its "full regalia", those forthcoming writings. Rather, it essays to summarize the principal features of their findings, those most salient for catalyzing deeper comprehension of the **F.E.D. «opus»**, especially for those still early in their encounters with it.

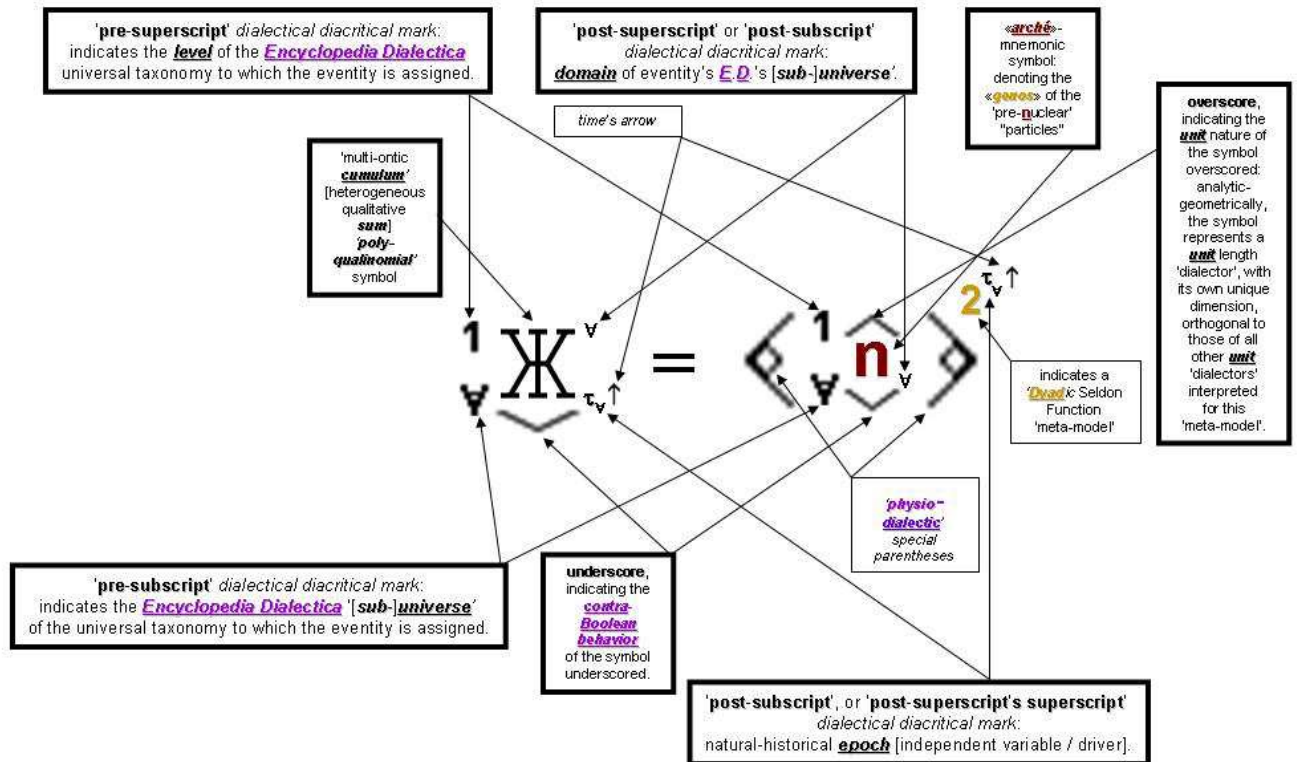
Dialectics *in general* is an instrument of **universal labor** [cf. Marx]. The *specific* instrument of universal labor central to this essay is the **F.E.D.** first dialectical arithmetic, and its 'contra-Boolean', algebraic dialectical logic, discovered by Karl H. Seldon, more than a decade ago. As documented especially in the initial volume of **A Dialectical "Theory of Everything"**, as well as elsewhere, this Seldonian "mathematics of dialectics" creates a capability to summarily model the total history of the known cosmos -- the total '**dialectic of nature**' [note the singular: the operative word is "dialectic", **not** "dialectic**s**"] -- by a single 'dialectical meta-equation'. This 'meta-equation' models that dialectic as a single, self-[re-]iterating, 'dialectically-fractal', universal self-movement, born from the '**self-«aufheben» self-meta-monadization**' of the «**monads**» of the «**arché-arithmos**», i.e., from the primordial population, of pre-**n**uclear "particles", of the «**genos**» $\frac{1}{\sqrt{n}}$, whose constituent «**species**» are the [non-hadronic] **bosons, leptons, & quarks**. This dialectic math. can achieve this feat with **20** symbolic elements, **or less**, viz. --

$$\frac{1}{\sqrt{n}} \left(\frac{1}{\sqrt{n}} \right)^{\sqrt{n}} = \left(\frac{1}{\sqrt{n}} \right)^{\sqrt{n}^{\sqrt{n}}}$$

Using the equation above, one can generate, by iterating it one 'epochal unit' beyond its model of the present, novel predictions regarding an expected future, next major emergence of new cosmological ontology. In the present essay, it is my task to apply this same central instrument of universal labor -- this dialectical algebra of the **n_Q** 'contra-Boolean' arithmetic -- to determine, & to present to you, in an optimally-ordered, intuitively-coherent, categorial-progression presentation, using this dialectical **method of** presentation, the contemporary 'ideo-totally' of the "Standard Arithmetics", those of the **N, W, Z, Q, R, & C** 'number-worlds', by means of a dialectical equation similar in form to the equation above. Ready or not, here we go!

The following page contains a JPEG graphic of the above-presented 'meta-equation', with its symbolic elements defined via call-outs.
Vignette #4 The Gödelian Dialectic of the Standard Arithmetics 4.4 - vi by M. Detonaciones of **Foundation Encyclopedia Dialectica** [F.E.D.]

Dyadic Seldon Function 'Meta-Model' for 'The Dialectic of Nature as a Whole': Its Symbolic Elements Defined via Callouts



The 'dialectical meta-equation' defined above describes the cumulative genesis -- the 'meta-genealogy' -- of every known kind of thing that presently exists in our universe, as of this 'meta-equation's' "'epoch'" $\tau_V = 8$. That is, it first reconstructs the *intra*-epochal "'self-evolution'", and the *inter*-epochal 'self-*meta*-evolution', of our cosmos, to date, and on into the future, as one vast, recurrent, cosmological «*aufheben*» process, in which the «*monads*» constituting each 'cosmo-«*arithmos*»' expandedly self-reproduce, and, at a critical point of their population self-concentration, give rise to the next higher «*monads*», constituting the next higher 'cosmo-«*arithmos*», via «*aufheben*» 'self-*meta*-«*monad*»-ization', a process which traces all the way back to primordial «*arithmos*», that of pre-nuclear "particles", denoted $1_V n_V$, as «*arche*»-«*arithmos*», and which culminates, as of $\tau_V = 8$, in a 'pre-construction' -- a prediction -- of the full irruption of a next, new kind of 'cosmo-ontology', one which is already "partially, "fractionally" emergent now, but mostly unnoticed as such.

By a 'meta-equation', we mean the «*aufheben*» 'meta-*monad*ization', or «*aufheben*» 'meta-*unit*ization', of an equation as «*monad*» or *unit*: a 'meta-equation' *unit* is a meta-*unit* of equations as *units*, such that each 'meta-equation' is "made up out of" a heterogeneous multiplicity of equation-*units*. The 'meta-equation' above is an 'equation-valued meta-equation' -- it "'contains'" a *qualitatively different* equation for every value of its independent variable, τ_V -- nine qualitatively distinct equations total for the range $\tau_V = 0$ through $\tau_V = 8$.

A Note about Notation. The **Encyclopedia Dialectica** notation and notational conventions, employed throughout this essay, are described in the modules reachable via the following links --

- [Module 1](#)
- [Module 2](#)
- [Module 3](#)
- [Module 4](#)
- [Module 5](#)

-- all of which are excerpted from the initial volume of **A Dialectical "Theory of Everything"**. Of the Modules listed above, the key Modules for this essay -- Modules **1, 2, 4, and 5** -- are reproduced below.

Some **Notational Conventions** Employed in this Main **Dialectical 'Meta-Model'**, and throughout this Exposition

$\overset{1}{\underset{\vee}{\hat{n}}}$ denotes the earliest known **kind of being**, or **ontological category**, extant in the «kosmos», per current knowledge; the **category** of sub-nuclear "particles", comprising the non-hadronic "particles" **sub-categories**, e.g., "**photons**", "**quarks**", "**electrons**", "**neutrinos**",... as distinct from that of "sub-atomic particles", comprising hadronic **sub-categories** [**mesons**, **protons**, **hyperons**, etc.], denoted $\overset{1}{\underset{\vee}{\hat{s}}}$.

$\overset{\wedge}{}$ The '**overscore**', above the **n**, signifies that the **ontological category** is itself treated, in this **meta-model**, as a '**unit**', or '**monad**', as it is, indeed, assigned to the \mathbb{T}_1 **unit** in the **un** interpreted arithmetic of the \mathbb{N} **dialectical language** used for this **dialectical meta-model**, notwithstanding that this **category** is "made up out of" **sub-categories sub-units**, each of which has, in turn, its own [sub-]sub-units...

$\underset{\vee}{}$ The '**underscore**', below the **n**, signifies that the **ontological category**, as arithmetical / algebraic **operator**, exhibits the characteristic that we call '**contra Boolean**', per the axioms of the \mathbb{N} **Dialectical Arithmetic**, i.e., that this operator, when "**squared**" -- when it operates **upon itself**-- yields up a product which, in **Boolean** fashion, still does reproduce itself, but which also yields, **non-amalgamatively added** to that "**Boolean**" component, one of **qualitatively different, new ontology**, a **new ontological category unit**, signifying **ontic self-negation**, but also **self-conservation**, & **self-elevation**, of that predecessor 'onto', thus exhibiting the «**aufheben**», or core **dialectical**, characteristic. For this case: 'sub-atomics' '**onto**' = **new part of self-«aufheben» self-product** of 'pre-nuclear' '**onto**', or:

$\overset{1}{\underset{\vee}{\hat{n}}}^2 = \overset{1}{\underset{\vee}{\hat{n}}} \overset{1}{\underset{\vee}{\hat{n}}} = \langle \overset{1}{\underset{\vee}{\hat{n}}} \oplus \overset{1}{\underset{\vee}{\hat{s}}} \rangle$, as each $\overset{1}{\underset{\vee}{\hat{s}}}$ "particle" is a **meta**-particle, made up out of a heterogeneous multiplicity of $\overset{1}{\underset{\vee}{\hat{n}}}$ "particles", thus is an «**aufheben**» **self-meta-«monad»-ization** by «**monads**» formerly constituent of the $\overset{1}{\underset{\vee}{\hat{n}}}$ **ontological category**.

$\overset{1}{\underset{\vee}{\mathbb{X}}}_1 = \langle \overset{1}{\underset{\vee}{\hat{n}}} \rangle^2 = \langle \overset{1}{\underset{\vee}{\hat{n}}} \oplus \overset{1}{\underset{\vee}{\hat{s}}} \rangle$, denotes a '**meta-state/epoch**' of the cosmological '**cumulum**' in which **both** the sub-nuclear "particles" **category** & that of the sub-atomic "particles", e.g., **protons**, may be extant.

Some **Notational Conventions** Employed in this Main **Dialectical Meta-Model**, and throughout [cont'd.]

$\overset{1}{\underset{\vee}{}}$ These 'pre-superscript' & 'pre-subscript' tags provide data as to an **onto's** "location" in the '**Human-Phenomic universe**' of discourse which the **Dialectical Meta-Model Equation** addresses. The '1' in ' $\overset{1}{}$ ' signifies that this **Model** addresses explicitly only the first, or most general, level or scale of the **Encyclopedia Dialectica Universal Taxonomy**; the ' $\underset{\vee}{}$ ' signifies that this **Meta-Model** is **Universal**, i.e., addresses **all** being that is even **possibly** manifest as of epoch τ , at that first, or "highest", **categoria| level**.

\uparrow This vertical, upwards-pointing arrow 'ideo-pictogram' signifies a monotonic **quantitative** escalation in the value of the variable whose symbol immediately precedes it. It invokes the **consecutive** 'Peanic' succession for that variable.

$\tau \uparrow$ This compound symbol signifies a monotonic, **consecutive** 'Peanic' escalation in the value of the purely-**quantitative** τ discrete-time epoch-variable, a variable remaining always within the **W** number-universe, **W** = { 0, 1, 2, 3, ... }.

\oplus The ideogram denoting the **dialectical-arithmetic** operation of **generalized addition**, for the **historical-dialectical** interpretive repertoire of the **Encyclopedia Dialectica** notation. This **addition** operation is "**non-amalgamative**" [cf. Musès] for '**dialectical meta-numbers**' of **unlike quality/ontology**, & is, on the contrary, '**super-amalgamative**', i.e., "**additively idempotent**", or "**non-quantitative**", for '**dialectical meta-numbers**' of like **quality/ontology**.

$\langle \rangle$ Ideogram denoting the "**open parenthesis**" grouping-operation for **purely-qualitative** '**dialectical meta-numbers**', interpreted as **purely-ontological category-values**, for the **historical-dialectical** interpretive repertoire of the **Encyclopedia Dialectica** notation.

$\rangle $ Ideogram denoting the "**close parenthesis**" grouping-operation for **purely-qualitative** '**dialectical meta-numbers**', interpreted as **purely-ontological category-values**, for the **historical-dialectical** interpretive repertoire of the **Encyclopedia Dialectica** notation.

$\overset{1}{\underset{\vee}{\mathbb{X}}}_\tau \uparrow$ Compound 'ideo-pictogram', denoting the ' \oplus ' -- **non-amalgamative / super-amalgamative** -- **sum**, or '**cumulum**', of the **ontological category units** possibly in existence as of the τ th epoch of **universe self-development**, for level **1** of the **Encyclopedia Dialectica Universal Taxonomy**, i.e., for the known «kosmos» as a **totality**: The '**Dialectical "Theory of Everything" Ontology**' as of 'Totality-epoch' τ .

Introduction. The *‘dialectical meta-model’* presented and narrated herein, generates, in an ideographical-shorthand, summary form, a ‘meta-systematic dialectical’ presentation of the “‘meta-system’” of the “Standard Arithmetics”. It does so in the logical, systematic-taxonomic optimal order of their growing complexity /- “‘thought-concreteness’” [or “‘features-richness’”], and in the order of presentation wherein each successor system is a “conservative extension” of, and thus *«aufheben»*-“contains”, all of its predecessor systems.

By a “‘meta-system’”, we mean an *«arithmos»*, or [mental] *assemblage*, of a heterogeneous multiplicity of systems, each involving, in part, qualitatively different [ideo-Jontology] relative to its predecessors, and that is ordered *diachronically*, as a ‘systems-progression’, progressing historically, even if the historical duration involved is one of “‘micro-history’”, spanning as little time as the duration of a single lecture, or of the reading of a single written presentation, i.e., even if the diachronic span is so brief as to make it ‘almost-synchronic’.

We adopt, herein, throughout, the *Encyclopedia Dialectica* notational convention of denoting, on the one hand, the *first-order-and-higher-order-logic* axioms-system of a given “number-space”, or “number-set”, denoted generically by **X**, by that same **X** with a single underscore, **X**, and of denoting, on the other hand, the *first-order-logic-only* axioms-system of that number-space, again, by that same **X**, but doubly-underscored: **X**. This notational convention embraces the irony of the ironic implication that a *first-order-logic-only* axiomatization of such a number-space is, in a certain sense, more than -- is broader than; is more general than -- its *first-order-plus-second-/higher-order-logic* axiomatization/specification; that **X**, though typically requiring far fewer axioms than **X**, is more encompassing than **X**, embracing more logical possibility than does **X**. This is crucially due to the fact that axioms-system **X** will “‘contain’” or imply “Non-Standard models” of the **X** *«arithmoi»*, models which are excluded by **X**. Thus, for one example, the **R** axiomatization of the space of the “Real” numbers, **R**, encompasses that “Non-Standard Model” of **R** which is Robinsonian “Non-Standard Analysis”, with its “actual infinitesimals” that help to ‘re-algebraicize’, and to simplify, many expressions of the integrodifferential calculus and of “Real Analysis”, bringing them closer to the more intuitive, founding forms expressed by Newton and Leibniz, but this time with the sanction of rigorous mathematical logic. Thus also, for another example, **N** encompasses the **N_Q**, dialectical, “Non-Standard Model” of the first-order Dedekind/Peano axioms -- a model which **N** excludes. The **N_Q** language is the language in which the central ideographical expressions of this essay are formulated.

[Note]: I render standard $i \equiv \sqrt{-1}$, herein, as **i**, and underscore also the symbols for the “qualitative units” [cf. Morris Kline] of the Quaternions and Octonions, in accord with the [non-standard] *Encyclopedia Dialectica* convention of underscoring the numerals that stand for numbers/operators/operations that exhibit the ‘contra-Boolean self-multiplication characteristic’, $n^2 \frac{1}{2} n \equiv [n^2 \neq n] \& [n^2 \neq n] \& [n^2 \neq n]$. I render “complex numbers” -- the “complex” or “‘intricate association or assemblage of [two] related things’” formed by non-amalgamatively adding, to an **a**-quantified **r**-unit, a **b**-quantified **i** unit -- here, in this context of ‘maximalized’ theoretical, semantical and syntactical explicitude regarding number-ideas, as **ar + bi**, or, even more ‘concept-explicitly’, as **ar + bri**, even though, given its “‘Boolean’” and ‘chameleonic’ character, the explicit presence of the **r** $\equiv +1$ unit has no computational-outcome effect, but only highlights a theoretical, conceptual issue of notational [a]symmetry. Also: Some of the unpublished *Encyclopedia Dialectica* manuscripts that I reviewed in preparation for writing this essay include, before the **n**-dimensional **K** “William Kingdon Clifford numbers”, the **16**-dimensional “Sedenions”, **S**, after the **8**-dimensional **O**ctonions, **O**, the **4**-dimensional **H**amilton Quaternions, **H**, the **2**-dimensional **C**omplex numbers, **C**, and the **1**-dimensional “Real” numbers, **R**, in the ‘meta-model’ presentation of the Standard Arithmetics. Others do not, holding **S** to be adequately contained in **K**. This issue is presently unresolved within the **E. D.** research community, so I have opted to present, herein, only the more ‘minimalist’ of the two alternatives.

As rendered in its unpublished manuscripts, the **E. D.** dialectical equation ‘meta-model’ for the systems-progression method of presentation of the “Standard Arithmetics” -- an *«aufheben»*-progression of *first-order-and-higher-order-logic* axioms-systems -- is solved therein [herein up to **C** only], as yielding the following *dialectical* progression of “conservative extensions” of such systems, as an ongoing ‘self-argument’/‘self-dialogue’, in **steps/** **stages** generically denoted by **s** --

$$\mathbf{N} \rightarrow \mathbf{W} \rightarrow \mathbf{Z} \rightarrow \mathbf{Q} \rightarrow \mathbf{R} \rightarrow \mathbf{C} \rightarrow \mathbf{H} \rightarrow \mathbf{O} \rightarrow \mathbf{K} \rightarrow \mathbf{G} \rightarrow \dots$$

$$\mathbf{s}: 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9 \quad 10$$

The ‘ \rightarrow ’ sign means “becomes”. With ‘ \equiv ’ for ‘definitional equality’, the other symbols used above are defined below --

N \equiv The higher[>]-than-first-order axiomatic system for the “Standard Arithmetic” of the “**Natural Numbers**”, **N** \equiv { **I, II, III, ...** };

W \equiv The >-than-first-order axioms-system for the Arithmetic of the “**Whole Numbers**”, **W** \equiv { **0, 1, 2, 3, ...** };

Z \equiv The >-than-first-order axioms-system for the “Standard Arithmetic” of the “**Integers**”, **Z** \equiv { ..., **-3, -2, -1, ± 0 , +1, +2, +3, ...** };

Q \equiv The higher-order system of the “Standard Rational Numbers”, **Q** \equiv { ..., **-3/+1, -2/+1, -1/+2, ± 0 /+1, +1/+2, +2/+1, +3/+1, ...** };

R \equiv The higher-order system of the “Standard **R**eal Numbers”, **R** \equiv { ..., **$-\pi, -e, -\sqrt{2}, \pm 0, \sqrt{2}, e, \pi, \dots$** };

C \equiv The higher-order system of the “Standard **C**omplex Numbers”, **C** \equiv { **Rr + Ri** }, wherein **r** \equiv +1.0..., **i** $\frac{\perp}{\perp}$, **1** +1, and **-1** = **i**² $\frac{\perp}{\perp}$ **i**;

H \equiv The higher-order system of the “**H**amilton *Quaternions*”, **H** \equiv { **Rr + Ri + Rj + Rk** }, | **i**² = **j**² = **k**² = **-1**, **-ij** = **ji**, **-jk** = **kj**,
such that **r** $\frac{\perp}{\perp}$, **1** $\frac{\perp}{\perp}$ **i** $\frac{\perp}{\perp}$, **1** $\frac{\perp}{\perp}$ **j** $\frac{\perp}{\perp}$, **1** $\frac{\perp}{\perp}$ **k**, and such that **-1** = **i**² $\frac{\perp}{\perp}$ **i**, and **-1** = **k**² $\frac{\perp}{\perp}$ **k**;

O \equiv The higher-order system of the “Cayley/Graves **O**ctonions/**O**ctaves”, **O** \equiv { **Rr + Ri + Rj + Rk + Rl + Rm + Rn + Ro** }, such that
r $\frac{\perp}{\perp}$ **i** $\frac{\perp}{\perp}$ **j** $\frac{\perp}{\perp}$ **k** $\frac{\perp}{\perp}$ **l** $\frac{\perp}{\perp}$ **m** $\frac{\perp}{\perp}$ **n** $\frac{\perp}{\perp}$ **o**, and such that **-1** = **l**² $\frac{\perp}{\perp}$ **l**, **-1** = **m**² $\frac{\perp}{\perp}$ **m**, **-1** = **n**² $\frac{\perp}{\perp}$ **n**, and **-1** = **o**² $\frac{\perp}{\perp}$ **o**;

K \equiv The higher-order system of the William **K**ingdon Clifford Arithmetics for “Geometric Algebra”, **K_{n-1}** \equiv { **Rr + Rk₁ + ... + Rk_{n-1}** }, e.g.,
such that **K₃** \ni **k₃** \equiv **r₀r + r₁k₁ + r₂k₂ + r₃k₃ + r₄k₁k₂ + r₅k₁k₃ + r₆k₂k₃ + r₇k₁k₂k₃**, & such that **r** $\frac{\perp}{\perp}$, **1** $\frac{\perp}{\perp}$ **k₁** $\frac{\perp}{\perp}$, **1** ... $\frac{\perp}{\perp}$, **1** $\frac{\perp}{\perp}$ **k_{n-1}**;

G \equiv The higher-order system of the Herman **G**rassmann Numbers’ “Arithmetic for **n**-dimensional Geometries”, **G** \equiv { { **Rg_{D,1} + ... + Rg_{D,n}** } }.

The underscored letters above signify a number unit, a “*qualitative* unit” [cf. Morris Kline], which instantiates the generic

‘contra-Boolean’ characteristic $\frac{\perp}{\perp} \frac{\perp}{\perp} \frac{\perp}{\perp}$, wherein the relation sign ‘ $\frac{\perp}{\perp}$ ’ signifies *non-quantitative*, or *qualitative, inequality*

[‘negated trichotomy principle’, ‘slash-negating’ all three of the standard, ‘>’, ‘=’, and ‘<’ relations]. The relation sign ‘ \perp ’ signs the “‘analytic-geometric’” mutual perpendicularity of the number-lines ‘unit-ed’ by the “qualitative units” so related: **r, i, j, k**, etc.

Although the ‘dialectical meta-equation meta-model’ presented herein is essentially a *synchronic*-slice, viewpoint-of-

present-day-mathematics-based, [Meta-]*Systematic Dialectic*, it contains hints, e.g., in the **N** $\frac{\perp}{\perp}$ **W** distinction, of the

Terran human [psycho]*history* of today’s “Standard Arithmetics”; of what would be front-and-center in a *diachronic*, “longitudinal”, [Psycho]*Historical-Dialectical* ‘meta-model’ of the actual chronological succession of *de facto* systems of arithmetic embraced by majorities of Terran mathematicians in a succession of historical epochs.

Such a [Psycho]*Historical-Dialectical* ‘meta-model’ of ‘The Meta-Evolution of Arithmetics’ is addressed to full regalia in the second volume of *Dialectical Ideography*, but is not addressed to any comprehensive degree herein.

In the present essay, we will follow this progression of axiomatic-system symbols, in full, only as far as **C**, the axioms-system of the “Standard Arithmetic” of the “**C**omplex Numbers” -- which is the most advanced arithmetical system in this sequence which is still well-known at least among the dwindling, remnant, high-school-algebra-conversant general public.