F.*E*.*D*. Vignette #11 --

‘‘‘Number Theory’’’,

Ancient vs. *Modern* vs. *Trans-Modern*.

by Aoristos Dyosphainthos

Author’s Preface. The purpose of **F.*E*.*D***. Vignette **#11** is to present **F.*E*.*D***.’s «***Arithmos***» ***Theory*** -- a ***psychohistorical dialectical synthesis*** of **Ancient** «***Arithmos***» ***Theory*** **&**/with **Modern** ‘‘‘**Number Theory**’’’ -- *without recourse to numbers*.

A Note about the On-Line Availability of Definitions of F.E.D. Key Technical Terms. Definitions of ***Encyclopedia Dialectica*** technical terms and ‘neologia’ are available on-line via the following URLs --

http://www.dialectics.org/dialectics/Glossary.html

https://www.point-of-departure.org/Point-Of-Departure/ClarificationsArchive/ClarificationsArchive.htm

-- by clicking on the links associated with each such term, listed, alphabetically, on the web-pages linked above.

The ***Encyclopedia Dialectica*** special terms most fundamental to this vignette are indicated below, together with links to their ***E.D***. definitions --

«***arithmos***» and «***arithmoi***»
http://point-of-departure.org/Point-Of-Departure/ClarificationsArchive/Arithmos/Arithmos.htm
https://www.point-of-departure.org/Point-Of-Departure/ClarificationsArchive/Arithmoi/Arithmoi.htm

«***aufheben***»
https://www.point-of-departure.org/Point-Of-Departure/ClarificationsArchive/Aufheben/Aufheben.htm

http://www.dialectics.org/dialectics/Glossary_files/F.E.D.,%20A%20Dialectical%20%27%27Theory%20of%20Everything%27%27,%20Volume%200.,%20FOUNDATIONS,%20Edition%201.00,%20first%20published%2010DEC2011,%20Definition,%20AUFHEBEN,%2018AUG2011,%20JPEG.jpg

‘***cumulum***’
http://point-of-departure.org/Point-Of-Departure/ClarificationsArchive/Cumulum/Cumulum.htm

«***monad***»
https://www.point-of-departure.org/Point-Of-Departure/ClarificationsArchive/Monad/Monad.htm

$_{N}$**Q** ***dialectical arithmetic/algebra***
http://www.dialectics.org/dialectics/Correspondence_files/Letter17-06JUN2009.pdf

‘***self-meta-monad-ization***’ or ‘***self-meta-individual-ization***’
http://point-of-departure.org/Point-Of-Departure/ClarificationsArchive/Meta/Meta.htm
http://point-of-departure.org/Point-Of-Departure/ClarificationsArchive/MetaMonadization/MetaMonadization.htm

-- we plan to expand these definitions resources as the **F.*E*.*D***. ***Encyclopedia Project*** unfolds.

⌶₁ ⟵ I. Ancient «*Arithmos*» *Theories*.

⊞

Exemplary Ancient, '***Quanto-Qualitative***' Definitions of '"***Number***"' as «***Arithmos***»: E.g., by Euclid and by Aristotle.

Not generally realized by we Moderns is the psychohistorical fact that the Ancients' concept of '''Number''' -- named, in ancient Greek, by the word «***Arithmos***», the ancient Greek word from which the modern English word "***Arithm***etic" descends -- as proven by the 'psychoartefacts' that the Ancients left behind, and that have come down to us, still extant, was qualitatively, '***ideo-ontologically***' ***different*** from our Modern concept of "Number".

Summarily we can say that Ancients defined an «***Arithmos***» as an '''***assemblage*** of qualitative -- ***of*** multiple-qualities-exhibiting -- ***units***/***things*** of a given, ***single kind***'''. «***Arithmoi***» are thus both 'quanto-qualitative', and sensuous and ideational, phenomena, not "purely quantitative", 'quantifier-only' ideo-phenomena, such as, e.g., **2**, **3**, **4**, **5**, **6**, and **7**, ...

Herein we will take, as representative of the Ancients' concept of '''Number''', in its philosophical and/or mathematically technical form, the still-extant recorded thoughts of Euclid and Aristotle.

For Euclid [third Century B.C.E.]: "Euclid defines in the *Elements*, VII, 2, a number as "the multitude made up of units" having previously (*Elements*, VII, 1) said that a unit is "that by virtue of which each of existing things is called one." As a unit is not composed of units, neither EUCLID nor ARISTOTLE regard a unit as a number, but rather as "the basis of counting, or as the origin [i.e., as the «***arché***» -- A.D.] of number." There is an echo of this Euclidean definition in CANTOR's definition of the cardinal number as a set composed of nothing but units ... ." [H. Hermes, *et al*., ***Numbers***, Springer Verlag, [NY: **1991**], p. **12**].

Note that this -- "self-evident?" -- claim that "a unit is not composed of units" posits a radical duality between multiplicity / number on the one hand, and unity on the other, i.e., between «***arithmos***» **&** «***monad***». The Ancients excepted a single «***monad***» from their category of '''*number*''' simply because a ***single*** «***monad***» is ***not*** an ***assemblage*** of «*monads*» -- is ***not*** plural.

This claim ignores the fact that reality is rife with 'assemblages of meta^{n+1}-***units***, each of which is composes of a sub-assemblage of meta^{n}-***units***' -- e.g., as a population with molecules as its ***units*** is one of each of whose ***units*** is composed of atoms as its sub-***units***, as a population with atomic nuclei as its ***units*** is one each of whose ***units*** is composed of "sub-atomic particles" [e.g., protons] as its sub-***units***, and as a population of "sub-atomic particles" is one each of whose ***units*** is composed of "pre-sub-atomic particles" [e.g., quarks and gluons] as its sub-***units***.

For Aristotle [circa **335** B.C.E]: "Apart from this definition of number, which is oriented towards the idea of counting, one can find in ARISTOTLE the following statement: that which is divisible into discrete parts is called [A.D.: «***plethos***»] πληθος (multitude), and the bounded (finite) multiplicity is called the number (ARISTOTLE [1], 1020a, 7.14). The [A.D.: Ancient] Greeks thus regarded as numbers, only the natural numbers, excluding unity; fractions were treated as ratios of [A.D.: "natural"] numbers, and irrational numbers as relationships between incommensurable magnitudes in geometry ... . [*Ibidem*].

Actually, the statement above is an anachronism, a '*moderno-morphism*', and a 'retro-projection' of the Modern meme of "number" back upon the Ancient one: the Ancients did not hold the modern conception of the "natural" number, as "***pure***, *unqualified* *quantifier*". On the contrary, as we shall show herein, via Diophantus's *circa* **250** C.E. treatise ***The Arithmetica***, the ancient meme of '''number''' was a hybrid, '*quanto-qualitative*' one.

Attending closely to the qualitative, 'ideo-ontological' distinction of the Ancient concept of '''number''' from the Modern can enable one to solve -- with both speed and clarity -- mysteries that still baffle many scholars of philosophy, e.g. --

"***arithmos***: number; ***arithmêtikê***: the science of number. Zero was unknown as a number and one also was not counted as a number, the first number being *duas* [A.D.: or '*dyos*'] -- two. From the Pythagoreans, *ton arithmon nomizontes arkhên einai* -- who consider number to be the first principle (Ar. Met. 986a15) -- number played a great part in metaphysics, especially in Plato's unwritten doctrines, involving obscure distinctions of e.g. *sumblêtoi* and *asumblêtoi* -- addible and non-addible numbers." [J. O. Urmson, The Greek Philosophical].

The «***Arithmoi Eide-tikoi***» of Plato's static, eternal ***dialectic***, or '***ideo-taxonomy***', were, in his conception, «***arithmoi***» of «***Eide-Monads***» -- ***Assemblages*** of «***Ιδεα***»-***Units*** -- for Plato's reified, deified «***Ιδεας***», which he supposed to be the immutable, perfect, Parmenidean Causes behind the imperfect copies of them which somehow constituted and conducted the dynamic flux of our sensuous world.

Per Plato, for each such Causal «***Ιδεα***», call it 'I_1', any ***perfect*** copy of ***I***t was ***redundant*** in terms of philosophical logic, and could not exist: $I_1 + I_1 \neq$ '$2I_1$'; instead, $I_1 + I_1 = I_1$ [an algebraic property which Modern mathematics names "additive idempotency"].

Moreover, for any two -- heterogeneous, qualitatively/ontologically distinct -- such «***Ιδεας***», call them I_1 and I_2, their very "***apples*** versus ***oranges***" heterogeneity makes them "***non***-amalgamative" [cf. Dr. Charles Musès] if added together:

$I_1 + I_2 \neq$ '$2I_1$', and $I_1 + I_2 \neq$ '$2I_2$'; instead,

$I_1 + I_2 = I_1 + I_2$, without further possibility of reduction within this language;

"apples plus oranges" equals "apples plus oranges", irreducibly so, at this level.

Thus, in both 'self-addition' and 'other-addition', Plato's «***Eide***»-***Units*** are "***un***addible", '***un***sum-able' -- «***a****sumblêtoi*».

Also, given that the *original* Pythagoreans held that «***arithmoi***» -- '''***assemblages*** of qualitative, multiple-qualities-exhibiting ***units***/***things*** of various ***single kinds***''', i.e., '''*populations of individual things* [*including of physical, sensuous things*]''' -- constitute reality, it is no longer sure that the *original* Pythagoreans were *raving idealist mystics*, as is so often presumed, based upon the 'retro-projection' of the modern meme of '''number''' upon their Ancient «***arithmos***» idea.

Ancient Alexandria's 'Proto-Renaissance', **&** Diophantus's '***Qualifier-Quantifier*** Proto-Algebra', at Dark Ages' Door.

The ***first*** known 'protoic' emergence of "symbolical algebra", as distinct from the already ancient 'prose algebra' -- or "rhetorical algebra" -- and of an algebra "symbolical" in the specific sense of '***ideo***gramic symbols', ***not*** exclusively of either '***picto***gramic symbols' and/or of '***phono***gramic ["***phon***etic"] symbols' [which, after all, would simply mean 'prose algebra', or "rhetorical algebra" again] -- was in a *circa* **250** C.E. work by Diophantus, entitled ***The Arithmetica***.

This text, ***The Arithmetica*** [«***Arithm****êtikê*»], taught the '''art''', or '''technology''' or '''technique(s)''' [«***tekhnê***»], or '''craft''', or '''skill''', or '''science''' of «***Arithm****oi*» in general.

This text developed an intermediate stage between "rhetorical" algebra and full-blown "symbolical", 'equational' algebra, which has often been termed "syncopated" [abbreviated] algebra, in which minimized abbreviations ["syncopations"] of words served as 'proto-ideogramic' symbols for arithmetical ***quant***ities, or '''***quant***ifiers''', ***and for arithmetical qualities or*** '''***qualifiers***''', including for equations involving ***un***known ***quant***ities which Diophantus showed how to "solve" -- how to systematically render the ***un***known ***quant***ities ***known***.

Diophantus's particular style of "abbreviation" or "syncopation" -- an unprecedented style as far as is known -- was, apparently, to take the first letter of the Greek word to be abbreviated, and to place atop that first letter the second Greek letter of that word. Thus, only two Greek letters -- the first two letters -- of the Greek word "survived" his abbreviation process. Known numerical values were expressed using single Greek letters, with a dash or a " prime" atop each letter, in ordinal correspondence [i.e., $\bar{\alpha} = \mathrm{I}$, $\bar{\beta} = \mathrm{II}$, $\bar{\gamma} = \mathrm{III}$, etc.], in accord with longstanding Ancient arithmetical tradition.

The context of this work by Diophantus was the Ancient Egyptian city of Alexandria, after the zenith of the extraordinary, unprecedented Human-Phenomic -- scientific, technological, and institutional -- developments there, that Karl Seldon has described as the Western "Proto-Renaissance". Diophantus's revolution in mathematics was cut short, in part, because it arose *circa* **250** C.E., just a few centuries before the tidal wave of the fall of the Roman Empire, and the undertow dragging Ancient Hellenistic civilization down into the hellish abyss of the European Dark Ages, smashed into Alexandria, suppressing this progressive trend, and delaying its resumption, continuation, and supersession for another ≈ ten centuries.

Regarding the mathematical aspect of this "Proto-Renaissance" in Ancient Alexandria, we find the following from the historical record: "The earliest attempt to found a university, as we understand the word, was made at Alexandria. ... It was particularly fortunate in producing within the first century of its existence three of the greatest mathematicians of antiquity -- Euclid, Archimedes, and Apollonius. They laid down the lines on which mathematics subsequently developed, and treated it as a subject distinct from philosophy: hence the foundation of the Alexandrian Schools is rightly taken as the commencement of a new era. Thenceforward, until the destruction of the city by the Arabs in 641 A.D. [i.e., C.E.], the history of mathematics centers more or less round that of Alexandria". [W. W. Rouse Ball, ***A Short Account of the History of Mathematics***, Dover [New York: **1960**], pp. **50**-**51**].

Howard Eves describes, as follows, the lead-up to the founding of Alexandria --

"The period following the Peloponnesian War was one of political disunity among the Greek states, rendering them easy prey for the now strong kingdom of Macedonia which lay to the north. King Philip of Macedonia was gradually extending his power southward and Demosthenes thundered his ***unheeded warnings***. The Greeks rallied too late for a successful defense and, with the Athenian defeat at Chaeronea in 338 B.C.[E.], Greece became a part of the Macedonian empire. Two years after the fall of the Greek states, ambitious Alexander the Great succeeded his father Philip and set out upon his unparalleled career of conquest which added vast portions of the civilized world to the growing Macedonian domains. Behind him, wherever he led his victorious army, he created, at well-chosen places, a string of new cities. It was in this way, when Alexander entered Egypt, that the city of Alexandria was founded in 332 B.C.[E.]. ... It is said that the choice of the site, the drawing of the ground plan, and the process of colonization for Alexandria were directed by Alexander himself. From its inception, Alexandria showed every sign of fulfilling a remarkable future. ***In an incredibly short time, largely due to its very fortunate location at a natural intersection of some important trade routes, it grew in wealth, and became the most magnificent and cosmopolitan center of the world***. ..." [Howard Eves, **An Introduction to the History of Mathematics** (**3**rd ed.), Holt, Rinehart **&** Winston (NY: **1969**), pp. **112**-**113** ***emphasis added*** by A.D.].

-- and the institutional innovations which seeded its unprecedented destiny --

"After Alexander the Great died in 323 B.C.[E.], his empire was partitioned among some of his military leaders, resulting in the eventual emergence of three empires, under separate rule, but nevertheless united by the bonds of the Hellenistic civilization that had followed Alexander's conquests. Egypt fell to the lot of Ptolemy. ... He selected Alexandria as his capital and, ***to attract learned men to his city, immediately began the erection of the famed University of Alexandria. This was the first institution of its kind. ... Report has it that it was highly endowed and that its attractive and elaborate plan contained lecture rooms, laboratories, gardens, museums, library facilities, and living quarters. The core of the institution was the great library, which for a long time was the largest repository of learned works to be found anywhere in the world, boasting, within forty years of its founding, over 600,000 papyrus rolls. It was about 300 B.C.[E.] that the university opened its doors and Alexandria became, and remained for close to a thousand years, the intellectual metropolis of the Greek race*** [and not of the Greek "race" alone, but of the Occidental Afro/Euro/Near-Asian hemisphere of humanity entire! -- A.D.]." [*Ibid*., p. **113**, ***emphasis added*** by A.D.].

In summary: "***No other city has been the center of mathematical activity for so long a period as was Alexandria from the days of Euclid (ca. 300 B.C.[E.]) to the time of Hypatia (A.D. 415 [C.E.])***. It was a very cosmopolitan center, and the mathematics that resulted from Alexandrian scholarship was not all of the same type. ..." [Carl Boyer, Uta Merzbach, ***A History of Mathematics*** (**2**nd edition), John Wiley **&** Sons, Inc. (NY: **1991**), p. **178**, ***emphasis added*** by A.D.].

Morris Kline well-describes the mathematical, technological, economic, and cultural momenta that converged into the genesis of the Ancient Alexandrian "Proto-Renaissance" in the following passages.

After the early death of Alexander, the Ptolemaic emperors of Egypt carried forward with Alexander's plans: "After his death ... the empire was split into three independent parts. ... Egypt, ruled by the Greek Ptolemy dynasty, became the third empire. Antigonid Greece and Macedonia gradually fell under Roman domination and became unimportant as far as the development of mathematics is concerned ... The major creations following the classical Greek period were made in the Ptolemaic empire, primarily in Alexandria."

"That the Ptolemaic empire became the mathematical heir of classical Greece was not accidental. The kings of the empire ... pursued Alexander's plan to build a cultural center at Alexandria. ... These rulers therefore brought to Alexandria scholars from all the existing centers of civilization and supported them with state funds."

"About 290 B.C.[E.] Ptolemy Soter built a center in which the scholars could study and teach. This building, dedicated to the ***muse***s, became known as the ***Muse***um, and it housed poets, philosophers, philologists, astronomers, geographers, physicians, historians, artists, and most of the famous mathematicians of the Alexandrian Greek civilization."

"Adjacent to the Museum, Ptolemy built a library, not only for the preservation of important documents but for the use of the general public. This famous library was said at one time to contain 750,000 volumes, including the personal library of Aristotle and his successor Theophrastus. Books, incidentally, were more readily available in Alexandria than in classical Greece because Egyptian papyrus was at hand. In fact, Alexandria became the center of the book-copying trade of the ancient world."

"The Ptolemies also pursued Alexander's plan of encouraging a mixture of peoples, so that Greeks, Persians, Jews, Ethiopians, Arabs, Romans, Indians, and Negroes came unhindered to Alexandria and mingled freely in the city. Aristocrat, citizen, and slave jostled each other and, in fact, the class divisions of the older Greek civilization broke down." [Morris Kline, ***Mathematical Thought from Ancient to Modern Times***, Volume **I**, Oxford University Press [New York: **1972**], pp. **101**-**102**, ***emphases*** ***added*** by A.D.].

Ancient Alexandria's favorable locus, with respect to the concentration and centralization of ancient commerce and wealth there, also contributed crucially to the consummation of its peoples' cultural ambitions: "The civilization in Egypt was influenced further by knowledge brought in by traders and by the special expeditions organized by the scholars to learn more about other parts of the world. Consequently, intellectual horizons broadened. The long sea voyages of the Alexandrians called for far better knowledge of geography, methods of telling time, and navigational techniques, while commercial competition generated interest in materials, in efficiency of production, and in improvement of skills. Arts that had been despised in the classical period were taken up with new zest and training schools were established. Pure science continued to be pursued but was also applied." [*Ibid*., pp. **102**-**103**].

Part of what resulted was ***an unprecedented flowering of engineering and technology****, even though not supported by strong incentives to apply this technology in production, given the still predominantly pre-capitalist, peasant-/serf-, 'artisanal-', and slavery basis of the prevailing social relations of production, especially after the Roman conquest of Egypt, in* ***31*** *B.C.E.*: "***The mechanical devices created by the Alexandrians were astonishing even by modern standards***. Pumps to bring up water from wells and cisterns, pulleys, wedges, tackles, ***systems of gears***, and ***a mileage measuring device no different from what may be found in the modern automobile*** were used commonly. ***Steam power was employed to drive a vehicle along the city streets in the annual religious parade***. Water or air heated by fire in secret vessels of temple altars was used to make statues move. ... Water power operated a musical organ and made figures on a fountain move automatically while compressed air was used to operate a gun. New mechanical instruments, including an improved sundial, were invented to refine astronomical measurements." [*Ibid*.; pp. **103**, ***emphases by*** A.D.].

The disparaging squeamishness and 'needlessness' of classical Greek "aristocratic" slave-holders with regard to '''hands-dirtying work''' ['''fit only for slaves'''] -- and with regard to practical and commercial applications of the fruits of intellectual labor -- was overcome in Ancient Alexandria: "Proclus, who drew material from Germinus of Rhodes (1st cent. B.C.[E.]), cites the latter on the divisions of mathematics...: arithmetic (our theory of numbers), geometry, mechanics, astronomy, optics, geodesy, canonic (science of musical harmony), and logistics (applied arithmetic). According to Proclus, Germinus says: The entire mathematics was separated into two main divisions with the following distinction: one part concerned itself with the intellectual concepts and the other with material concepts." Arithmetic and geometry were intellectual. The other division was material. However, the distinction was gradually lost sight of ... ***One can say***, ***as a broad generalization***, ***that the mathematicians of the Alexandrian period severed their relation with philosophy and allied themselves with engineering***." [*Ibid*., pp. **104**-**105**, ***emphases by*** A.D.].

Hero[*n*] *of Alexandria, and his teacher, Ctesibius [who may have been responsible for the "Antikythera Mechanism"], incarnate this mathematico-technological momenta of the Ancient Alexandrian 'Proto-Renaissance'*: "Proclus refers to ***Heron*** as *mechanicus*, which might mean a mechanical engineer today, and discusses him in connection with *the inventor* ***Ctesibius***, *his teacher*. ***Heron*** was also a good surveyor. ... The striking fact about ***Heron***'***s*** work is his commingling of rigorous mathematics and the approximate procedures and formulas of the Egyptians. On the one hand, he wrote a commentary on Euclid, used the exact results of ***Archimedes*** (indeed he refers to him often), and in original works proved a number of new theorems of Euclidean geometry. On the other hand, he was concerned with applied geometry and mechanics and gave all sorts of approximate results without apology. He used Egyptian formulas freely and much of his geometry was also Egyptian in character. ..."

"His applied works include *Mechanics*, *The Construction of Catapults*, *Measurements*, *The Design of Guns*, *Pneumatica* (the theory and use of air pressure), and *On The Art of Construction of* ***Automata***. He gives designs for water clocks, measuring instruments, ***automatic machines***, weight lifting machines, and war engines." [*Ibid*., pp. **116-117**, ***emphases added*** by A.D.].

Factors in the demise of this Ancient Alexandrian "Proto-Renaissance" are described, by Howard Eves, as follows --

"The city of Alexandria enjoyed many advantages, not the least of which was long-lasting peace. During the reign of the Ptolemies, which lasted for almost 300 years, the city, although on occasion beset with internal power struggles, remained free from external strife. This was ended by a short period of conflict when Egypt became part of the Roman empire ... The closing period of ancient times was dominated by Rome. ... The economic structure ... was essentially based on agriculture, ***with a spreading use of slave labor***. ***The eventual decline of the slave market, with its disastrous effect on Roman economy***, ***found science reduced to a mediocre level***. The Alexandrian school gradually faded, along with the breakup of ancient society. [*op. cit.*, p. **164**, ***emphases added*** by A.D.].

-- and --

"Greek science reached its pinnacle at Alexandria ... The decline was caused by a combination of technological, political, economic, and social factors. ... ***The Romans used slave labor to an almost unprecedented degree, especially after the founding of the Empire*** by Augustus in 31 B.C.[E.]. ***More than half of the Empire's inhabitants were slaves***. With slaves to do most of the backbreaking work, there was little perceived need for labor-saving devices, such as the pulleys and levers invented by Archimedes ... hence, scientists had little incentive to invent them." [*Op. cit.*, pp. **137-138**, ***emphases added by*** A.D.].

-- and by Morris Kline thusly --

"The fate of ***Hypatia***, an Alexandrian mathematician of note and the daughter of *Theon of Alexandria* [the redactor of Euclid's ***Elements*** -- A.D.], symbolizes the end of the era. ***Because she refused to abandon the Greek religion, Christian fanatics seized her in the streets of Alexandria and tore her to pieces***. ... From the standpoint of the history of mathematics, the rise of Christianity had unfortunate consequences. Though the Christian leaders adopted many Greek and Oriental myths and customs with the intent of making Christianity more acceptable to converts, *they opposed pagan learning and ridiculed mathematics, astronomy, and physical science; Christians were forbidden to contaminate themselves with Greek learning*. Despite cruel persecution by the Romans, Christianity spread and became so powerful that the ***emperor Constantine*** (272-337 [C.E.]) was obliged to consign it a privileged position in the Roman Empire. *The Christians were now able to effect even greater destruction of Greek culture. The* ***emperor Theodosius*** *proscribed the pagan religions and, in 392 [C.E.] ordered that the Greek temples be destroyed.* ***Pagans were attacked and murdered throughout the empire***. *Greek books were burned by the thousands. In that year* ***Theodosius*** *banned the pagan religions, the Christians destroyed the temple of Serapis* [in Alexandria -- A.D.], which still housed the only extensive collection of Greek works. *It is estimated that 300,000 manuscripts were destroyed. Many other works written on parchment were expunged by the Christians so that they could use the parchment for their own writings* ... *In 529 [C.E.], the Eastern Roman emperor Justinian closed all the Greek schools of philosophy, including* ***Plato's Academy***. ... The final blow to Alexandria was the conquest of Egypt by the upsurging Moslems in ... 640 [C.E.]. *The remaining books were destroyed* on the ground given by ***Omar***, the Arab conqueror: "Either the books contain what is in the Koran, in which case we do not have to read them, or they contain the opposite of what is in the Koran, in which case we must not read them." *And so for six months the baths of* ***Alexandria*** *were heated by burning rolls of parchment*. After the capture of ***Alexandria*** by the Mohammedans, the majority of the scholars migrated to Constantinople, which had become the capital of the Eastern Roman Empire. Though ***no activity along the lines of Greek thought could flourish in*** *the unfriendly Christian atmosphere of Byzantium*, this flux of scholars and their works to comparative safety ***increased the treasury of knowledge that was to reach Europe eight hundred years later***. ***It is perhaps pointless to contemplate what might have been***. ***But one cannot help observe that the Alexandrian Greek civilization ended its active scientific life on the threshold of the modern age***. ***It had the unusual combination of theoretical and practical interests that proved so fertile a thousand years later***. ***Until the last few centuries of its existence, it enjoyed freedom of thought, which is also essential to a flourishing culture***. ***And it tackled and made major advances in several fields that were to become all-important in the Renaissance***: quantitative plane and solid geometry; trigonometry; algebra; calculus; and astronomy." [*Op. cit.*, pp. **180-181**, ***emphases*** *added* ***by*** A.D.].

It is in the above-described '''*psychohistorical*''' context that the work of Diophantus of Alexandria can be comprehended -- as a ***hybrid*** product of waning Hellenistic memes, and of a 'protoic', precocious, prevenient partial prefigurement of core components of the as yet unborn Human Phenome of Modernity.

Morris Kline assesses the work of Diophantus in the following terms:

"The highest point of ***Alexandrian Greek algebra*** *is reached with* ***Diophantus****.* ... His work towers above that of his contemporaries; ***unfortunately, it came too late to be highly influential in his time because a destructive tide was already engulfing the civilization****.* ***Diophantus*** wrote several books that are lost in their entirety. ... His great work is the ***Arithmetica*** which, ***Diophantus*** says, comprises thirteen books. We have six [**6** *surviving in Greek, that is*; **4** *more were recently found, in Arabic, possibly translations into Arabic of* ***Hypatia****'s Greek commentaries on books* **4** *through* **7**, *rather than of* ***Diophantus****' originals* -- A.D.] ... *One of* ***Diophantus****' major steps is* ***the introduction of symbolism*** [i.e., of ***proto-ideography*** -- A.D.] ***in algebra****.* ... The appearance of such ***symbolism*** is of course remarkable but ***the use of powers higher than three*** *is even more extraordinary. The classical Greeks could not and would not consider a product of more than three factors* ***because such a product had no*** [then-recognized -- A.D.] ***geometrical significance*** [i.e., given the apparently **3**-and-no-more/no-less-dimensional physical space of our world -- A.D.]. *On a* ***purely arithmetical basis****, however, such products do have a meaning; and this is precisely the basis* ***Diophantus*** *adopts*." [*Op. cit.*, pp. **138**-**139**, ***emphases added*** *by* A.D.].

Diophantus symbolized a[ny], generic number, in a ***dual*** format, as a juxtaposition -- a '''product''', in effect -- of two semantic '''co-factors''', called, by Karl Seldon, an '''arithmetical ***qualifier***''', **&** an '''arithmetical ***quantifier***''', viz. --

$$\overset{o}{\mathrm{M}}\,\varsigma$$

-- with the "syncopated" ***unit qualifier*** symbol $\overset{o}{\mathrm{M}}$ signifying the «**Mo-nad**», the generic, abstract [and '***quant***ifiable'] "***unit***", or '''***one***-ness''', standing generically and indifferently for any specific kind of ***unit*** -- e.g., for an ontological ***unit***, or for a metrical ***unit***, or even for an undifferentiated combination of the two.

Examples include a ***unit*** of the "***kind of thing***" category -- or '''***ontological*** category''' -- of the ***quality*** of "apple-ness", i.e., an ***apple unit***, or an ***orange unit***, or a ***pound unit*** as "***unit*** *of* ***measure***" or "***metrical unit***", or the ***combined***, ***undifferentiated unity of a metrical and an ontological quality unit***, e.g., "a pound of apples", or "a pound of oranges".

The symbol ς, is the generic ***quantifier*** symbol, often used by Diophantus to represent the unknown, and to-be-solved-for, value in one of Diophantus's 'proto-algebraic proto-equations'. This number symbol is drawn, as was typical in Ancient Greek '''*logistics*''' [practical arithmetic], from the Greek alphabet. It is the version of the Greek letter sigma, **σ**, that is used when sigma is the final letter of a Greek word, e.g., in particular, ς is the last letter of the Greek word «***arithmos***», or «***αριθμος***». In modern English, it coincides with the final **s**, i.e., with English letter suffix that signifies ***plurality***.

Thus, the expression above might stand, indifferently, for the prose representations "six apples" [or, literally, "apples six" -- ***qualifier*** first, or in first place, followed by ***quantifier*** second, or in second place], or "six oranges", or "six pounds", or "six pounds of apples", etc.

That is, Diophantus, in keeping -- for the most part -- with **Ancient** «***Arithmos***» Theory, does ***not*** symbolize number in general simply as --

$$\varsigma$$

-- i.e., as an abstract, "pure" ***quant***ifier, ***without qual***ification, as would be the case if Diophantus had ***followed*** -- i.e., if he had ***anticipated*** -- the ***meme*** of European Renaissance humanity, ***after*** the world-historic '***Elision of the Qualifiers***'.

This world-historic '***Elision***' was brought about, '''***psychohistorically***''', we hold, in the post-Dark-Ages European Human Phenome -- which was also the point-of-origin of the [***psycho***]***historically-specific Capitalist Phenome***, or «***mentalité***», by the intensive practice of the capital-relation by so much of the population: of the monies-[capitals-]mediated exchanges of commodities[-capitals], that emerged, in the lead-up to the Western European Renaissance, as a far more intensive such praxis than was ever reached within the socio-economic limitations of Ancient Mediterranean times, and of their substantially slavery-based mode of social production.

This 'capital-praxis' was captured, in its purest, simplest essence -- abstracting from its more concrete determinations, involving mediation by money [price] and by production processes, outside of the process of circulation of capitals, by Marx's «***arché***» for «***Das Kapital***» as a whole, "***The Elementary or Accidental Form of Value***", set forth by Marx from the ***beginning*** of that work, in Vol. **I**, Part **I**, Chapter ***I***., Section **3**.**A**. of, as the ***systematic-dialectical*** '***seed cell***' of that entire work, and expressed by Marx, in his 'algebraic/rhetorical' notation, in the form of the '''*exchange-equations*''' --

x commodity A = y commodity B

or, e.g., as: 20 yards of linen = 1 coat

-- and, later, by Seldon, as --

$$\{ \mathbf{c}_j\underline{\mathbf{C}}_j \fallingdotseq \mathbf{c}_k\underline{\mathbf{C}}_k \},$$

with **c**ommodity ***quant***ifiers $\mathbf{c}_j \gtrless \mathbf{c}_k$, ***despite*** that the **C**ommodity ***qual***ifiers $\underline{\mathbf{C}}_j \neq \underline{\mathbf{C}}_k$.

. . .

II. Modern "Number Theories".

[*Forthcoming*].

III. F.E.D.'s Seldonian, Trans-Modern -- Modern/Ancient Hybrid -- «Arithmos» Theories.

We of F.E.D. use the word '''***number***''' in a far more concrete sense than has become habitual in the **Modern** World, and, in certain ways, with a sense much more like it had in the **Ancient** World.

In our '''***Number Theory***''', as a modernization of the ancient '«***Arithmos***»-***Theory***', or '«***Monads***»- ***Theory***', '''***number***''' means ***not*** an abstract, "pure" ***quantity*** as such, as per those "***number***" conceptions so central to the **Modern** [***un***consciously, experientially ***law-of-capital-value-inculcated***] «***mentalité***».

On the contrary, in our usage, ***number*** means something far closer to sensuous 'empiricality'.

It refers to a specific multiplicity of ***units***/***individuals***/***monads***, akin to a plural but finite "***population***" of the ***individuals*** of the same ***kind***, such that each ***individual*** is a concrete, determinate, 'multi-***qual***itative' ['multi-***qual***ity'], attributes-rich [ev]entity, ***not*** a distilled, rarefied mental abstraction of "pure, ***unqual***ified ***quant****ity*".

In such a usage, '''***numbers***''' thus no longer differ only ***quant***itatively: such '''***numbers***''' have different "***kinds***".

And, Old '''***numbers***''' create New '''***numbers***''': they not only expand themselves ***quant***itatively, as ***populations*** of their ***units***, but ***qual***itatively, ***ontologically*** as well.

That is, Old ***kinds*** of '''***numbers***''' create New ***kinds*** of '''***numbers***''' by means of '***self-meta-monad-ization***', that is, via '*self-meta-**unit**-ization*', or '*self-meta-**individual**-ization*'.

The process of ***self-meta-monad-ization***' is a ***self***-«***aufheben***» ***process***, which is to say, a ***dialectical process***.

A theory of the progressive self-construction of our cosmos -- in the form of a single, recurrent, mounting, cumulative, helical '***dialectic of nature***' -- can be constructed on the basis of noticing that, e.g. --

The [self-changing]'''***number***''' [cosmological ***population***] of pre-nuclear "particles" [e.g., of non-Hadronic, "non-composite" bosons and fermions, such as quarks] created the [dynamical, "fluent" [cf. Newton] self-changing]'''***number***''' of sub-atomic "particles" [e.g., of primordial protons and neutrons], by their own '***self-meta-monad-ization***';

The [self-changing]'''***number***''' [cosmological ***population***] of sub-atomic "particles" [e.g., non-Hadronic, "non-composite" bosons and fermions, such as, quarks] created the [self-changing/other-changed/other-changing]'''***number***''' of [ionic] atomic nuclei [e.g., primordial Deuterium, Tritium, Helium, and Lithium], by their '***self-meta-monad-ization***';

The [self-changing]'''***numbers***''' [galactic ***populations***] of atomic nuclei created the [dynamical, "fluent", self-changing/other-changed/other-changing]'''***numbers***''' of molecules [e.g., of galactic "inter-stellar medium" accumulating H_2, O_2, CN, H_2O, CO_2, CH_4, etc.], by their own brand of such '***self-meta-monad-ization***';

The [dynamical, "fluent", self-changing]'''***numbers***''' [cosmological ***populations***] of molecules created the [self-changing/other-changed/other-changing]'''***numbers***''' of 'pre-eukaryotic' living cells, by their own, natural-***historically-specific*** «***species***» of '***self-meta-monad-ization***';

etc.

Our Marxian, immanent critique of both the **Modern** and the **Ancient** conceptions of '''***Number***''' find their foundation in the '''psychohistorical''' insights, into both the **Modern** and the **Ancient** human ideologies -- into the **Modern** versus the **Ancient** 'Human Phenomes' -- embodied in Marx's immanent, dialectical critique of capitalist political economy.

In his "***Elementary Form of Value***", Marx discovered ***much more momentous*** than even the ultimate "***seed***" ***category*** -- the «***arché***» ***category*** -- from which there '''***descends***''', in an '***ideo-meta-genealogical***', ***dialectical method-of-presentation*** sense, the rest of his entire, vast, comprehensive critique of the political economy of capital; of the capital social-relation-of-production; of the capitals ***social system*** of global, prehistoric humanity.

He also discovered the universal ***un***conscious paradigm of '***The Modern*** «***mentalité***»', and of its most characteristic symptom -- the "***purely quantitative***" frame of mind, and '***The Elision of the Qualifiers***' from conception, from perception, and from mathematical -- starting especially with arithmetical -- symbolic expression.

Marx therein and thereby discovered the ***secret***, not just of "***The German Ideology***", but of the total, global, human "***Modern Ideology***" entire -- of the total '***Human Phenome***' of a planetary humanity that embodies and incarnates Capital.

We of **F**.***E***.***D***. have found working with the $_{\mathbf{N}}\mathbf{Q}$ arithmetic/algebra, as with its successor systems, to be a worthwhile and cognitively ***healing*** practice for we **F**.***E***.***D***. monastics.

In working with the $_{\mathbf{N}}\mathbf{Q}$, one is working with '''***numbers***''' that are ***purely qualitative***.

A given, generic $\boxed{\;}_{k}$ is interpreted, or ***speci***fied, as "standing for" an «***arithmos***», a ***number***, in part, in the **Ancient** sense: as "standing for", in effect, an ***ontological category*** representing the ***speci***al 'common-***kind***-ness' that unites all of the ***individuals***; that all of the «***monads***» which inhere in that ***ontological category*** share, like the '''in-tension''' of an '''ex-tension''', i.e., of a "set of elements".

The generic symbol $\boxed{\;}_{k}$, for a $\mathbf{k}$ in $\mathbf{N}$, thus ***interpreted***, means a ***number*** of ***in***definite/changing ***cardinality***, creating a kind of Marxian version of the "intentional" variables of the original Boolean algebra.

The practice of the expression of experienced/experimented reality, using the language of the $_{\mathbf{N}}\mathbf{Q}$ ***numbers***, is, we find, a liberating "spiritual practice" -- in the sense of a Marxian version of Hegel's "Objective Spirit": of '***The Human Phenome***'.

That is, this activity of ours is a ***healing*** modifier of our individual human phenomes, one that lifts us beyond the collective, 'ideologized' "Mind", the typical «***mentalité***», of our time -- beyond the "Mind" of '***The Modern Ideology***'; beyond the '***Money Mind***', beyond the one-sidedly, ***purely-quantitative*** «***mentalité***», the "Mind" of "***The Elementary Form of*** [***Commodity***] ***Value***" as ***un***conscious ***universal paradigm*** -- in short, beyond 'the capital-value «***mentalité***»'.

This practice thereby helps us to free our minds to ***see*** in new and wider ways -- to think beyond the blockages characteristic of '***The Modern Ideology***', the ideology of capital-value as *supreme* value, or even as *only*-value.

If you believe that such ***seeing*** is a part of ***your*** life path, then we commend this practice also to ***you***.

Links to definitions of additional ***Encyclopedia Dialectica*** special terms deployed in the discourse above --

«***arché***»
https://www.point-of-departure.org/Point-Of-Departure/ClarificationsArchive/Arche/Arche.htm

Boole's Algebra
http://point-of-departure.org/Point-Of-Departure/ClarificationsArchive/BoolesAlgebra/BoolesAlgebra.htm

categorial
http://point-of-departure.org/Point-Of-Departure/ClarificationsArchive/Categorial/Categorial.htm

category
http://point-of-departure.org/Point-Of-Departure/ClarificationsArchive/Category/Category.htm

dialectical categorial progression
http://point-of-departure.org/Point-Of-Departure/ClarificationsArchive/CategorialProgression/CategorialProgression.htm

'''***eventity***'''
https://www.point-of-departure.org/Point-Of-Departure/ClarificationsArchive/Eventity/Eventity.htm

ontological category
http://point-of-departure.org/Point-Of-Departure/ClarificationsArchive/CategoryOntological/CategoryOntological.htm

ontology
https://www.point-of-departure.org/Point-Of-Departure/ClarificationsArchive/Ontology/Ontology.htm