Discovering **W**hole-Qualifier Space ($_{\mathbf{w}}\underline{\mathbf{Q}}$) via **W**-Cum Space ($\underline{\mathbf{C}}_{\mathbf{w}}$)

(OR: *What a difference including an "origin-element" makes!*)

by
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About this Brief: The purpose of this F.E.D.* "Brief #6" is to extend the **N**-**C**um ($\underline{\mathbf{C}}_{\mathbf{N}}$) and $_{\mathbf{N}}\underline{\mathbf{Q}}$ co-discoveries by invoking an "origin" element ($\mathbf{C_0} = \mathbf{q_0}$) for their spaces, to obtain the **W**-**C**um ($\underline{\mathbf{C}}_{\mathbf{W}}$) and $_{\mathbf{W}}\underline{\mathbf{Q}}$ spaces. Surprisingly, this *origin qualifier* is like both **0** (under +) and **1** (under ×) in the **W**hole Numbers! The inclusion of $\mathbf{C_0}$ then becomes the basis for expanding this "**W**hole-**Q**ualifier" space to the co-discovery of "Integer-**Cum**ulation" and "Integer-**Q**ualifier" spaces (the topic of our next brief). Here, the *topic qualifier* $\mathbf{q_0}$ is seen as a unique "Boolean" **q**ualifier, which may be "assigned" to represent a "topic" ontology. An appendix explores ways of quantifying the "definiteness of **q**ualifiers," with $\mathbf{q_0}$ being regarded as the "least definite" **q**ualifier.

Overview of "Brief #5" and the "Origin" 'Cumulum' or qualifier

In F.E.D. Brief #5, "Discovering **N**atural-**Q**ualifier Space ($_{\mathbf{N}}\underline{\mathbf{Q}}$) via **N**-**Cum** Space ($\underline{\mathbf{C}}_{\mathbf{N}}$)", an exponential-type of isomorphism is used to map the **N**aturals onto **N**-**Cum** ($\underline{\mathbf{C}}_{\mathbf{N}}$), a "**C**umulation space" of idea set-numbers. Using the qualo-differential operator on elements of this space, we "co-discovered" **N**atural-**Q**ualifier space ($_{\mathbf{N}}\underline{\mathbf{Q}}$) and its addition and multiplication rules, or "axioms". This process of co-discovery answered several "early questions" often asked by both other F.E.D. readers, and by this author.

This process is now extended by asking a further question: *"Can we construct a cumulation, $\boldsymbol{X}$, such that, for any* $\mathbf{n}$ *in* $\mathbf{N}$: $\boldsymbol{X} + \underline{\mathbf{C}}_{\mathbf{n}} = \underline{\mathbf{C}}_{\mathbf{n}}$*?", or "Can we construct an $\boldsymbol{X}$, such that* $\boldsymbol{X} = \underline{\mathbf{C}}_{\mathbf{n}} \sim \underline{\mathbf{C}}_{\mathbf{n}}$*?"* (If so, would $\boldsymbol{X} = \underline{\mathbf{C}}_{\mathbf{n}} \sim \underline{\mathbf{C}}_{\mathbf{n}} = \{\,\}$, the null-set? See last subsection of **Appendix A2**.)

In Brief #5 we defined $\underline{\mathbf{q}}_{\mathbf{n}}$ as the differential of the **n**th **cum**ulation:

$$\underline{\mathbf{q}}_{\mathbf{n}} := \underline{\partial\mathbf{C}}_{\mathbf{n}} := \underline{\mathbf{C}}_{\mathbf{n}} \sim \underline{\mathbf{C}}_{\mathbf{n-1}} \quad \Leftrightarrow \quad \underline{\mathbf{C}}_{\mathbf{n}} = \underline{\mathbf{C}}_{\mathbf{n-1}} + \underline{\partial\mathbf{C}}_{\mathbf{n}} = \underline{\mathbf{C}}_{\mathbf{n-1}} + \underline{\mathbf{q}}_{\mathbf{n}},$$

and defined $\underline{\partial\mathbf{C}}_{\mathbf{1}} := \underline{\mathbf{C}}_{\mathbf{1}}$. This implies that, for $\mathbf{n = 1}$, we'd have $\underline{\mathbf{C}}_{\mathbf{1}} = \mathbf{C_{1\text{-}1}} + \underline{\partial\mathbf{C}}_{\mathbf{1}} = \mathbf{C_0} + \underline{\mathbf{C}}_{\mathbf{1}}$. So, for $\mathbf{n = 0}$, $\mathbf{C_0}$ (*without* underline) would be such an $\boldsymbol{X}$. Therefore, we postulate that such an $\boldsymbol{X} = \mathbf{C_0}$ exists, and that its differential is defined as itself (as also for $\mathbf{n = 1}$, $\underline{\partial\mathbf{C}}_{\mathbf{1}} = \underline{\mathbf{C}}_{\mathbf{1}}$):

*"Origin-**Cum**/**Q**ualifier" Existence Postulate*: There exists an *origin* (or *null*) **cum***ulation,* $\mathbf{C_0} := (\underline{\mathbf{C}}_{\mathbf{1}})^{\mathbf{0}}$, or *origin* (or *null*) *qualifier,* $\mathbf{q_0} := \underline{\partial}\mathbf{C_0} := \mathbf{C_0}$, less "definite" than $\underline{\mathbf{q}}_{\mathbf{1}}$, and such that $\mathbf{C_0} + \underline{\mathbf{C}}_{\mathbf{n}} = \underline{\mathbf{C}}_{\mathbf{n}}$, for every $\mathbf{n}$ in $\mathbf{N}$.

Using our linear $\underline{\partial}$ on $\mathbf{C_0} + \underline{\mathbf{C}}_{\mathbf{n}} = \underline{\mathbf{C}}_{\mathbf{n}}$, we see that it follows immediately that $\underline{\partial}\mathbf{C_0} + \underline{\partial\mathbf{C}}_{\mathbf{n}} = \underline{\partial\mathbf{C}}_{\mathbf{n}}$, or that $\mathbf{q_0} + \underline{\mathbf{q}}_{\mathbf{n}} = \underline{\mathbf{q}}_{\mathbf{n}}$, since $\underline{\mathbf{q}}_{\mathbf{k}} := \underline{\partial\mathbf{C}}_{\mathbf{k}}$ for any $\mathbf{k}$ in $\mathbf{N} \cup \{\mathbf{0}\}$. Thus, our implied addition of **q**ualifiers now allows another "amalgamated sum" besides $\underline{\mathbf{q}}_{\mathbf{n}} + \underline{\mathbf{q}}_{\mathbf{n}} = \underline{\mathbf{q}}_{\mathbf{n}}$, namely $\underline{\mathbf{q}}_{\mathbf{n}} + \mathbf{q_0} = \underline{\mathbf{q}}_{\mathbf{n}} = \mathbf{q_0} + \underline{\mathbf{q}}_{\mathbf{n}}$.

We now can expand both **N**-**Cum** ($\underline{\mathbf{C}}_{\mathbf{N}}$) and $_{\mathbf{N}}\underline{\mathbf{Q}}$ by appending this new element to each, just as is done when expanding the "**N**atural Numbers" to the "**W**hole Numbers":

In **Quantitative** space, The **Whole Numbers** $:= \mathbf{W} := \{0\} \cup \mathbf{N} = \{0, 1, 2, 3,\ldots\}$;

In **C̲umulation** space, The **W-C̲um**ulations $:= \underline{C}_W := \{C_0\} \cup \underline{C}_N = \{C_0, \underline{C}_1, \underline{C}_2, \underline{C}_3,\ldots\}$;

In **Q̲ualifier** space, The **Whole Q̲ualifiers** $:= {}_W\underline{Q} := \{q_0\} \cup {}_N\underline{Q} = \{q_0, \underline{q}_1, \underline{q}_2, \underline{q}_3,\ldots\}$.

We now study the interaction of the null-**Cum**ulation C_0 and the null-**q**ualifier q_0 with other elements in their respective spaces, under their respective addition and multiplication operations. Figure **1** illustrates the "appending" of this element, $C_0 = q_0$, to these spaces.

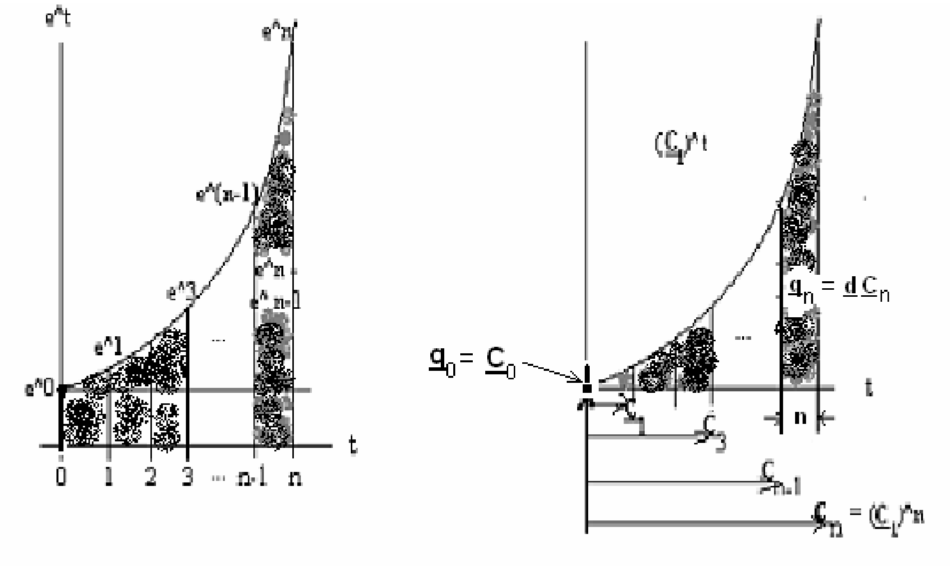
We first extend our isomorphism (used in Brief #5) to $\underline{C}_W$: $\mathbf{W} \rightarrow \underline{C}_w$, where $\mathbf{exQ(w)} := (\underline{C}_1)^w := \underline{C}_w$ for all $\mathbf{w}$ in $\mathbf{W}$, and where $\mathbf{exQ(0)} := (\underline{C}_1)^0 := C_0$.
We then observe that C_0 under the extended "**Cum** $\times$" rule behaves as **0** does under **+** in the **W**holes:

$$\mathbf{exQ(w+0)} = (\underline{C}_1)^w \times (\underline{C}_1)^0 = \underline{C}_w \times C_0 = \underline{C}_{w+0} = \underline{C}_w, \text{ for all } \mathbf{w} \text{ in } \mathbf{W}, \text{ \& for all } \underline{C}_w \text{ in } \underline{C}_W.$$

Thus, $C_0 = \mathbf{id(Cum} \times\mathbf{)}$, or *the "origin-cumulum", is the multiplicative identity* for $\underline{C}_W$!

> C_0 *<u>as a "Line/Point Cumulation</u>"*: In Brief #5, each $\underline{C}_n := \mathbf{exQ(n)} := (\underline{C}_1)^n$ was viewed as a "cumulative qualitative area" analogous to "quantitative area under e^t on the $\mathbf{t}$ interval $[0, n]$", or $\underline{C}_n$ was viewed simply as the image of the point $\mathbf{n}$ of $\mathbf{N}$, just as we now assume C_0 to be the image of point **0** of **W**. Yet, in Figure **1** we depict the **Cum**ulation C_0 as a "**1-D** line-segment, or **0-D** point" rather than as a "**2-D** area". Why? Briefly: To answer, we might instead think of each $\underline{C}_n$ as the image of [part-open] interval $(0, n]$ of the **R**eals, rather than of the point $\mathbf{n}$. In this way, for $n > 0$, $\underline{C}_n$ is a **2-D** image of a **1-D** interval $(0, n]$, but C_0 is a "**1-D** line-segment" image of a "**0-D** closed point interval": $[0, 0]$. In our next brief, our *origin* **C̲um**, C_0, will be used as a pivot "point" or "line", one that is also the origin to another **C̲um** space, viz., the "opposite" or "complementary" **C̲um**s to the **C̲um**s of $\underline{C}_N$. The reader may also regard this "point/line" aspect as a temporary reason why C_0 is <u>not</u> underlined while the other $\underline{C}_n$ (for $n > 0$) are <u>underlined</u>; the real reason is given in the *<u>Note on</u>* $C_0 = q_0$ *<u>not being underlined</u>* (below).

<u>Figure **1**</u>: Appending of $C_0 = q_0$ to form the **W-Cum** ($\underline{C}_W$) and ${}_W\underline{Q}$ spaces.



Next, we examine $\mathbf{C_0}$'s behavior under $\underline{\mathbf{C}}_{\mathbf{W}}$'s extended $+$:

$$\underline{\mathbf{C}}_w + \mathbf{C_0} = \underline{\mathbf{C}}_{max\{w,0\}} = \underline{\mathbf{C}}_w, \text{ for all } \mathbf{w} \text{ in } \mathbf{W}, \text{ i.e., for all } \underline{\mathbf{C}}_w \text{ in } \underline{\mathbf{C}}_{\mathbf{W}}.$$

Thus, $\mathbf{C_0} = \mathbf{id(Cum +)}$, or *the "origin-cumulum" is* *<u>also</u>* *the additive identity* for $\underline{\mathbf{C}}_{\mathbf{W}}$!

That "$\mathbf{id(Cum \times)} = \mathbf{C_0} = \mathbf{id(Cum +)}$" is an amazing result, because such a result is *impossible* in the mathematical system that is called an "algebraic field", such as the **Q**uotient numbers (Fractions) or the **R**eal numbers, or even the **C**omplex numbers for that matter! As proven here, in **Appendix A1**, such a result is possible if and only if '$\mathbf{A + A = A}$ for all $\mathbf{A}$ in $\mathbf{S}$, an *associative*, *distributive* **<u>S</u>**ystem with $\mathbf{id(+)}$, $\mathbf{id(\times)}$, and $\mathbf{[-id(\times)]}$', as is the case in both the space of the **W**-**<u>C</u>um**s ($\underline{\mathbf{C}}_{\mathbf{W}}$), and in the ${}_{\mathbf{W}}\underline{\mathbf{Q}}$ space.

We now confirm that "$\mathbf{id(\times)} = \mathbf{q_0} = \mathbf{id(+)}$" for the extended $+$ and $\times$ on the ${}_{\mathbf{W}}\underline{\mathbf{Q}}$ space elements:

$$\mathbf{q_0} + \underline{\mathbf{q}}_w = \underline{\mathbf{q}}_w, \text{ for every } \mathbf{w} \text{ in } \mathbf{W}, \text{ and for every } \underline{\mathbf{q}}_w \text{ in } {}_{\mathbf{W}}\underline{\mathbf{Q}} \text{ (\textit{shown earlier})},$$

and, for all $\mathbf{w}$ in $\mathbf{W}$, and for all $\underline{\mathbf{q}}_w$ in ${}_{\mathbf{W}}\underline{\mathbf{Q}}$, we have both products [commutative *in this case*]:

$$\underline{\mathbf{q}}_w \times \mathbf{q_0} = \mathbf{q_0} + \underline{\mathbf{q}}_{0+w} = \mathbf{q_0} + \underline{\mathbf{q}}_w = \underline{\mathbf{q}}_w$$

and

$$\mathbf{q_0} \times \underline{\mathbf{q}}_w = \underline{\mathbf{q}}_w + \underline{\mathbf{q}}_{w+0} = \underline{\mathbf{q}}_w + \underline{\mathbf{q}}_w = \underline{\mathbf{q}}_w.$$

Thus, $\mathbf{id(\times)} = \mathbf{q_0} = \mathbf{id(+)}$, or *the "origin* **q***ualifier" is* *<u>both</u>* *additive* **&** *multiplicative identity element* in ${}_{\mathbf{W}}\underline{\mathbf{Q}}$!

<u>Note on</u> "<u>**OQ$_\mathbf{W}$**" space</u>: It can also be shown (as is done in the appendices of Brief #4) that $\mathbf{q_0}$ is <u>also</u> the *additive* and *multiplicative identity element* in all of **W**-based **<u>O</u>**pen **<u>Q</u>**ualifier space, "$\underline{\mathbf{OQ}}_{\mathbf{W}}$", the set or space of all finite sums (and products) of elements from **W**hole-numbers **<u>q</u>**ualifier space (${}_{\mathbf{W}}\underline{\mathbf{Q}}$). Furthermore, it can be shown that because of the behavior of its only "amalgamative sums" with its otherwise "*<u>non</u>*-amalgamative sums", the $+$ operation is "associative" in $\underline{\mathbf{OQ}}_{\mathbf{W}}$, but $\times$ is not. Mathematically, this says that $\underline{\mathbf{OQ}}_{\mathbf{W}}$ is "closed" under $+$ **&** $\times$, and that $\mathbf{<}\underline{\mathbf{OQ}}_{\mathbf{W}}, \mathbf{+>}$ is a "commutative monoid." Although $\underline{\mathbf{OQ}}_{\mathbf{W}}$ is operationally "closed" for the $+$ **&** $\times$ operations, we regard $\underline{\mathbf{OQ}}_{\mathbf{W}}$ as "open" in another sense -- "open" to countless possible "interpretations" of any product or sum in its many modeling applications!

<u>Note on</u> $\mathbf{C_0 = q_0}$ *<u>not being underlined</u>*: The reader may have appropriately asked, "Why isn't either $\mathbf{C_0}$ or $\mathbf{q_0}$ <u>underlined</u>, as are the other **<u>C</u>um**ulations and **<u>q</u>**ualifiers?" Underlining of **<u>C</u>um**ulations and **<u>q</u>**ualifiers indicates their "*contra*-Boolean" nature, i.e., that $\underline{\mathbf{q}}_n \times \underline{\mathbf{q}}_n \neq \underline{\mathbf{q}}_n$ for all $\mathbf{n}$ of $\mathbf{N}$. However, for $\mathbf{C_0}$, and for $\mathbf{q_0}$, this <u>*not*</u> the case, since $\mathbf{C_0 \times C_0 = C_{0+0} = C_0}$, and $\mathbf{q_0 \times q_0 = q_0 + q_{0+0} = q_0 + q_0 = q_0}$. So, $\mathbf{C_0}$ and $\mathbf{q_0}$ are "Boolean": $\mathbf{C_0^2 = C_0}$ and $\mathbf{q_0^2 = q_0}$. Hence neither is underlined.

$\mathbf{q_0}$: Its Meaning, Interpretation, Name; Interpreting $\mathbf{q_n + q_n = q_n}$ & $\mathbf{q_n - q_n = ?}$

But what might $\mathbf{q_0}$ mean in terms of any assigned "ontology"? We again consult our quantitative analog, zer**0**. Zero is $\mathbf{id(+)}$ for the **W**hole numbers: $\mathbf{n + 0 = n}$ for every $\mathbf{n}$ in $\mathbf{N}$. Zer**0** represents **0** units of some "topic unit" for the topic to which its number space has been assigned, e.g., **0** units "of apple" in a number space assigned to represent apples. In essence, zer**0** names the "essence or topic quality **x**" in a "space of **x**s". Actually, it may be easier to identify **0**'s successor, **1**, as representing, e.g., "**1** apple unit" in a "space of apples".

Similarly, with **q**ualifiers, it may be easier to identify $\mathbf{q_0}$'s successor, $\underline{\mathbf{q_1}}$, as representing "the *first* "kind of being" (ontology) for a *genus* of apple **q**ualities" in a "space of 'apple' *species* [kinds of being]". Then, $\mathbf{q_0}$ would represent "the *essence/topic* of $X =$ 'apple kinds of being'," or *the 'null ontology' for* $X =$ *apples*. Just as **0** represents **0** units of some *topic-unit* (e.g., $\mathbf{x} =$ apples), so does $\mathbf{q_0}$ represent *the topic-essence* of ontology X (e.g., about apples). In either case, to this writer it seems appropriate to label $\mathbf{q_0}$ as the "origin **q**ualifier", but it also seems a terrible misnomer to use the term "null **q**ualifier" in reference to the essential purpose of such a "noble number"!

However, $\mathbf{q_0}$ ***is*** a "null **q**ualifier" in that has "null effect", under $+$ or $\times$, upon any other **q**ualifier, and may appear to be like the "null set". In any sum, $\mathbf{q_0} + \underline{\mathbf{U}} = \underline{\mathbf{U}} + \mathbf{q_0} = \underline{\mathbf{U}}$; in any product, $\mathbf{q_0} \times \underline{\mathbf{U}} = \underline{\mathbf{U}} \times \mathbf{q_0} = \underline{\mathbf{U}}$, so $\mathbf{q_0}$ seems to say: "Yes, I recognize yo**U** as being of the same topic ontology, therefore I support yo**U** and always let yo**U** be yo**U** in any interaction with me." (At least I *thought* I heard $\mathbf{q_0}$ whispering that to one of **U**!)

Because each name below sheds a different light/shade-of-meaning, this author uses/accepts all of the names below as names for '$\mathbf{q_0}$':

origin qualifier, *zeroth qualifier*, *topic qualifier, essence qualifier, null-qualifier, null-ontology!*

Next, we observe that Zer**0** is the *only* quantitative number having the property $\mathbf{0} + \mathbf{0} = \mathbf{0}$, whereas *every* **q**ualifier has that property: $\underline{\mathbf{q_n}} + \underline{\mathbf{q_n}} = \underline{\mathbf{q_n}}$. Zer**0** (or any **q**ualifier) cannot qualitatively augment itself, or "aggrandize itself", via $+$. In its subtractive form: $\mathbf{0} - \mathbf{0} = \mathbf{0}$, Zer**0** can "give herself away" and still be her **W**hole **0**-self. Is not this true for ideas? -- *Giving away an idea, one* (*as an "idea-creator"/"idea-interceptor"*) *can still hold it!* Thus, *sharing an idea, we still retain it!* Since each $\underline{\mathbf{q_n}}$ is like an idea-set, it's no surprise that $\underline{\mathbf{q_n}} + \underline{\mathbf{q_n}} = \underline{\mathbf{q_n}}$. This property reveals each $\underline{\mathbf{q_n}}$ capable of producing endless copies of itself within the one amalgamative sum that it is! In essence, each $\underline{\mathbf{q_n}}$ is "a potential infinity in a finite '''one'''!"

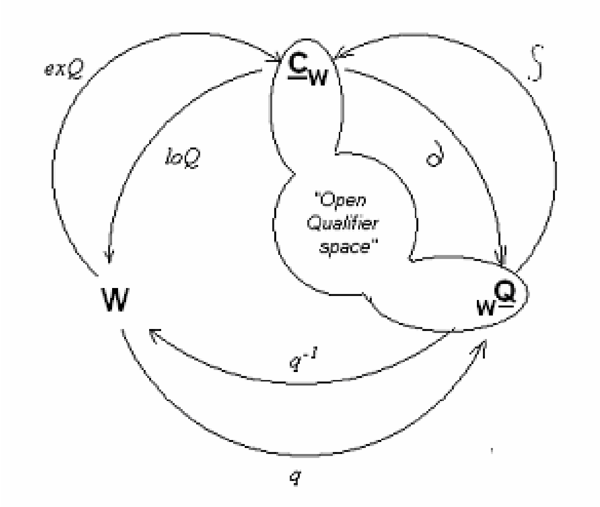
Finally, we may inquire into $_{\mathbf{w}}\underline{\mathbf{Q}}$ "subtractivity": Is $\underline{\mathbf{q_n}} - \underline{\mathbf{q_n}} = \underline{\mathbf{q_n}}$, or is $\underline{\mathbf{q_n}} - \underline{\mathbf{q_n}} = \mathbf{q_0}$?

Or, equivalently, is $\underline{\mathbf{C_n}} - \underline{\mathbf{C_n}} = \underline{\mathbf{C_n}}$, or is $\underline{\mathbf{C_n}} + \underline{\mathbf{C_{(-n)}}} = \mathbf{C_0}$? We shall address such key questions in our next brief, as we attempt to construct possible "Integer **Cums**", and "Integer **Q**ualifiers", under their $\times$ and $+$ operations.

Updating Relationships

Figure **2** summarizes the functional relationships among **W**, $\underline{\mathbf{C_w}}$, and $_{\mathbf{w}}\underline{\mathbf{Q}}$, via function inverses $\mathbf{q(\,)}$ and $\mathbf{q^{-1}(\,)}$, $\mathbf{exQ(\,)}$ and $\mathbf{loQ(\,)}$, and $\underline{\partial}\mathbf{(\,)}$ and $\underline{\int}\mathbf{(\,)}$, as was done in F.E.D. Brief #5. It also depicts "**O**pen **Q**ualifier space" as containing both the $\underline{\mathbf{C_w}}$ space and the $_{\mathbf{w}}\underline{\mathbf{Q}}$ space, since "**OQ** space" is the space of all possible sums (including 'idempotent' or 'single-element' sums), and products, which arise from $_{\mathbf{w}}\underline{\mathbf{Q}}$ **q**ualifiers under addition and multiplication. (Indeed, perhaps "**OQ** space" really is "like a bunny rabbit's head" because: "There are a lot of 'sums' between the bunny's two ears!" Enough to constitute a monoid.)

Figure **2**: Relationships of **W**, $\underline{\mathbf{C}}_{\mathbf{W}}$, and $_{\mathbf{W}}\underline{\mathbf{Q}}$ via **q( )**, $\mathbf{q}^{-1}$**( )**, **exQ( )**, **loQ( )**, $\underline{\partial}$**( )**, and $\underline{\int}$**( )**.



Appendix A1, herein, proves that, in an associative and distributive algebraic **S**ystem **S**, with **id(+)**, **id(×)**, and **[–id(×)]**, **A + A = A** (for every **A** in **S**), implies that **id(×) = id(+)**, and *vice versa*. In **Appendix A2**, also herein, we speculate about the nature of --

$\underline{\mathbf{C}}_{\mathbf{n}} := (\underline{\mathbf{C}}_{\mathbf{1}})^{\mathbf{n}} := \underline{\mathbf{C}}_{\mathbf{1}} \times \underline{\mathbf{C}}_{\mathbf{1}} \times \ldots \times \underline{\mathbf{C}}_{\mathbf{1}}$ (**n** times)

-- as a number under **Cum** × multiplication, including about the nature of --

$\{\underline{\mathbf{C}}_{\mathbf{n}}\} := \{\underline{\mathbf{C}}_{\mathbf{1}}\}^{\mathbf{n}} = \underline{\mathbf{C}}_{\mathbf{1}} \{\times\} \underline{\mathbf{C}}_{\mathbf{1}} \{\times\} \ldots \{\times\} \underline{\mathbf{C}}_{\mathbf{1}}$ (**n** "set-crosses", forming sets of "ordered **n**-tuples").

This construct has implications for the origin **q**ualifier, and also suggests ways to begin quantifying the "definiteness of **q**ualifiers," with $\mathbf{q}_{\mathbf{0}}$ being regarded as the "least definite" **q**ualifier.

Summary

By simply "appending" a "zer**0**-th" element ($\mathbf{C}_{\mathbf{0}} = \mathbf{q}_{\mathbf{0}}$), to the **N**-**Cum** ($\underline{\mathbf{C}}_{\mathbf{N}}$) and $_{\mathbf{N}}\underline{\mathbf{Q}}$ spaces, each is extended (along with their × and + operations) to obtain the **W**-**Cum** ($\underline{\mathbf{C}}_{\mathbf{W}}$) and $_{\mathbf{W}}\underline{\mathbf{Q}}$ spaces. In "**O**pen **W**hole-**Q**ualifier" space, $\underline{\mathbf{OQ}}_{\mathbf{W}}$, this origin element is shown to behave both similarly to the way that quantitative "**0**" (under +) does, and also similarly to the way that quantitative "**1**" (under ×) does, in **W**hole number space. Thus, $\mathbf{q}_{\mathbf{0}}$ is a unique "Boolean **q**ualifier", which serves as the "topic/essence **q**ualifier" for ontologies represented by, or assigned to, the $_{\mathbf{W}}\underline{\mathbf{Q}}$.

Next, $\mathbf{C}_{\mathbf{0}} = \mathbf{q}_{\mathbf{0}}$ becomes the basis for expanding these "**W**hole-**Q**ualifier" spaces, and for co-discovering "Integer-**Cum**ulation space" and "Integer-**Q**ualifier space" (the topics of our next brief), answering questions such as: *"What is* $\underline{\mathbf{C}}_{(-\mathbf{n})} + \underline{\mathbf{C}}_{\mathbf{n}}$ *?"*

And I first thought that we weren't adding much to ***Q****ualifier spaces by adding* $\mathbf{q}_{\mathbf{0}}$.

Was I ever mistaken!

-- *Joy-to-You* (July, 2012)

Appendix A1 -- Proof that '$id(\times) = id(+)$' in a System S $\Leftrightarrow$ '$A + A = A$' for any A in S

Let $<S, \times, +>$ be an *associative* algebraic system wherein $\times$ *distributes* over $+$, and wherein an $id(+)$, $id(\times)$, and an $[-id(\times)]$ exist, such that $-id(\times)$ is an *additive inverse* for $id(\times)$, the *multiplicative identity* element in **S**. The following then holds:

Theorem: $id(\times) = id(+) \Leftrightarrow A + A = A$ for any **A** in **S**.

█*Proof for*: $id(\times) = id(+) \Rightarrow A + A = A$ for any **A** in **S**.
The existence and definition of $id(+)$ imply that, *in particular*, $id(+) + id(+) = id(+)$, for $A = id(+)$ in **S**.
Then, for every **A** in **S**, we have, given that $id(\times) = id(+)$:
$A = A \times [id(\times)]$ [by definition of $id(\times)$] =
$A \times [id(+)]$ [substituting $id(+)$ for $id(\times)$] =
$A \times [id(+) + id(+)]$ [by definition of $id(+)$] =
$A \times [id(+)] + A \times [id(+)]$ [given '*distributivity*' of $\times$ over $+$] =
$A \times [id(\times)] + A \times [id(\times)]$ [substituting $id(\times)$ for $id(+)$] = [by definition of $id(\times)$]:
$A + A = A$ [by '*transitivity*']. **Q.E.D.**

Proof for: $A + A = A$ for any **A** in **S** [given $-id(\times)$] $\Rightarrow id(\times) = id(+)$.
For the special case $A = id(\times)$, the given *general case* $A + A = A$ implies that:
$id(\times) + id(\times) = id(\times)$, since $A + A = A$ too for $A = id(\times)$ in **S**,
$\Rightarrow \{id(\times) + id(\times)\} + [-id(\times)] = id(\times) + [-id(\times)]$, by adding $[-id(\times)]$ to both sides of the equation given immediately above,
$\Rightarrow id(\times) + \{id(\times) + [-id(\times)]\} = id(\times) + [-id(\times)]$, by 're-associating' LH side **{**sum terms**}** [given '*associativity*'],
$\Rightarrow id(\times) + id(+) = id(+)$, given $id(\times) + [-id(\times)] := id(+)$,
$\Rightarrow id(\times) = id(+)$, by definition of $id(+)$, applied to the LH side of the equation immediately above. **Q.E.D.** █

Appendix A2 -- "Is '$\underline{C}_n$' a 'number', a 'set', and/or a 'set-number'?" █ Answer: "All of the above!" █

In this appendix, we speculate about the nature of the "number $\underline{C}_n$" defined under "$\underline{C}$um $\times$" multiplication:

$$\underline{C}_n := (\underline{C}_1)^n := \underline{C}_1 \times \underline{C}_1 \times \ldots \times \underline{C}_1 \text{ (n times)},$$

versus the nature of the "set $\underline{C}_n$", or $\{\underline{C}_n\}$, under "set-cross" ["Cartesian Product"] multiplication, '$\{\times\}$':

$$\{\underline{C}_1\}^n = \underline{C}_1 \{\times\} \underline{C}_1 \{\times\} \ldots \{\times\} \underline{C}_1 \text{ (n "set-crosses").}$$

Let us, for a moment at least, redefine $\underline{C}_n$ as connoting *both* its set and its number aspects.
First, we have the iterative "differential" and "additive" forms:

$$\underline{q}_n := \partial\underline{C}_n := \underline{C}_n \sim \underline{C}_{n-1} \Leftrightarrow \underline{C}_n = \underline{C}_{n-1} + \partial\underline{C}_n = \underline{C}_{n-1} + \underline{q}_n.$$

<u>Key role for $\partial\underline{C}_n$ and the "set-cross" product rule:</u>
So, let's first (and rather "naturally") define the differential increment, $\partial\underline{C}_n := \underline{C}_n \sim \underline{C}_{n-1}$, as:

$$\underline{q}_n := \partial\{\underline{C}_n\} := \partial\{\underline{C}_1\}^n := \partial\{\underline{C}_1 \{\times\} \underline{C}_1 \{\times\} \ldots \{\times\} \underline{C}_1\}$$ (n "set-cross multiplications" yielding "ordered n-tuples").

Then let's define the entire "set-number", $\underline{C}_n$, as:

$$\underline{C}_n := \underline{C}_{n-1} \text{ union } \partial\{\underline{C}_1\}^n := \underline{C}_{n-1} \{+\} \partial\{\underline{C}_n\} := \underline{C}_{n-1} \{+\} \partial\{\underline{C}_1\}^n.$$

This "construct" is chosen because it uses the "subtractive difference equation", $\partial\underline{C}_n := \underline{C}_n \sim \underline{C}_{n-1}$, to define the differential increment, $\partial\underline{C}_n := \underline{q}_n$, in terms of the "set-cross" product, $\{\underline{C}_1\}^n$. (This process seems to impart an "automatic complexification" of the initial

$\underline{\mathbf{C}}$**um**-ontology ($\underline{\mathbf{C}}_1$) set into an "ordered $\mathbf{n}$-tuple" of itself (a kind of "autokinesis"?)! Then, to define the actual "set-number", $\underline{\mathbf{C}}_n$, we "add back" the previous $\underline{\mathbf{C}}_{n-1}$, thus ensuring its "subsumption" into $\underline{\mathbf{C}}_n$! So, using '$\langle\ldots\rangle$' as our notation for "ordered $\mathbf{n}$-tuples", if --

$\underline{\mathbf{C}}_1 := \{\, \mathbf{p}$ finite 'logical **e**lements' $\} := \{\, e_1, e_2, \ldots, e_p \,\}$ with $\mathbf{p} = \mathbf{p}^1$ logical elements, then we obtain a set of "ordered pairs", i.e., of 'ordered $\mathbf{2}$-tuples' --

$$\{\underline{C}_1\}^2 = \{ e_1, e_2, \ldots e_p \} \{\times\} \{ e_1, e_2, \ldots e_p \} = \{ \langle e_1, e_1\rangle, \langle e_1, e_2\rangle, \langle e_1, e_3\rangle, \ldots, \langle e_1, e_p\rangle \}$$

union

$$\{ \langle e_2, e_1\rangle, \langle e_2, e_2\rangle, \langle e_2, e_3\rangle, \ldots, \langle e_2, e_p\rangle \}$$

union ... union

$$\{ \langle e_p, e_1\rangle, \langle e_p, e_2\rangle, \langle e_p, e_3\rangle, \ldots, \langle e_p, e_p\rangle \}.$$

So, the "order" or "size" of $\{\underline{C}_1\}^2$ is that of $\#[\,\{\underline{C}_1\}^2\,] = p^2$.

Next:

$\{\underline{C}_1\}^3 = \{ \langle x_1, x_2, x_3\rangle$, such that each x_k is in $\underline{C}_1 = \{e_1, e_2, \ldots e_p\}$, for each $\mathbf{k}$ in $\{1, 2, 3\}\}$, so that the "order" of $\{\underline{C}_1\}^3$ is that of $\#[\,\{\underline{C}_1\}^3\,] = p^3$.

Or, generally:

$\{\underline{C}_1\}^n = \{ \langle x_1, x_2, x_3, \ldots, x_n\rangle$, such that each x_k is in $\underline{C}_1 = \{e_1, e_2, \ldots e_p\}$, each $\mathbf{k}$ in $\{1, 2, 3, \ldots, n\}\}$, with $\{\underline{C}_1\}^n$ having $\#[\,\{\underline{C}_1\}^n\,] = p^n$ elements.

Exploring measures of individual **q**ualifier "definiteness": $\underline{\mathrm{Def}}(\;)$ and $\log_p\underline{\mathrm{Def}}(\;)$.

Such a construct allows us to begin quantifying the "definiteness of **q**ualifiers", using the "order ($\#(\;)$)" of the differential set $\underline{q}_n := \{\underline{C}_1\}^n$. Let us re-label our function $\#(\;)$ as $\underline{\mathrm{Def}}(\;)$, a qualo-function from **q**ualifier space $\{\underline{q}_n\}$ into quantitative space $\{p^n\}$:

$$\underline{\mathrm{Def}}(\underline{q}_n) := \#(\underline{q}_n) = \#(\{\underline{C}_1\}^n) = p^n, \text{ or simply: } \underline{\mathrm{Def}}(\underline{q}_n) := p^n,$$

(a quantitative measure of $\underline{q}_n$*'s "***q***ualitative definiteness").*

This means that $\underline{\mathrm{Def}}(\;)$ can be used to "naturally" define the "**q**ualitative ordering" ($\dashv\!\!-$) of $\{\underline{q}_n\}$ in terms of the quantitative ordering ($<$) in $\{p^n\}$:

$$\underline{q}_k \dashv\!\!- \underline{q}_m \iff \underline{\mathrm{Def}}(\underline{q}_k) < \underline{\mathrm{Def}}(\underline{q}_m) \iff p^k < p^m \iff k < m \quad [\text{given } p > 1].$$

Under this definition, q_0 is regarded as "least definite", with a *"definiteness of* q_0" $= \underline{\mathrm{Def}}(q_0) = \underline{\mathrm{Def}}(\{\underline{C}_1\}^0) = p^0 = 1 < p^k$, given $p > 1$. This, in turn, implies that the "initial ontology", $\underline{q}_1$, via its "interpretation", must consist of at least two logical elements (alternatives?; "intra-duals"?), otherwise $p = 1$, and every **q**ualifier, $\underline{q}_n$, would have equal "unitary definiteness", since $\underline{\mathrm{Def}}(\underline{q}_n) = p^n = 1^n = 1$.

$\underline{\mathrm{Def}}(\;)$ is only one such quantitative measure of **q**ualitative definiteness. But, to us (to me at least), $\underline{\mathrm{Def}}(\;)$ seems, intuitively, to give "too large a value" for this "definiteness", as it increases exponentially, as p^n. Perhaps we desire a "more moderate", or "linear" (in $\mathbf{n}$), measure. This is easily accomplished by simply taking $\log_p(\;)$ of $\underline{\mathrm{Def}}(\underline{q}_n)$:

$$\log_p(\underline{\mathrm{Def}}[\underline{q}_n]) := \log_p(p^n) = n, \text{ or as one composite "}\log_p\underline{\mathrm{Def}}(\;)\text{" function:}$$

$$\log_p\underline{\mathrm{Def}}(\underline{q}_n) := n.$$

Actually, the triple composite function, "$\log_p\underline{\mathrm{Def}\partial}(\;)$" ($\log_p\underline{\mathrm{Def}}(\;)$ acting after the operator $\underline{\partial}(\;)$ acts on a $\underline{C}_n$) maps the set of whole $\underline{\mathbf{C}}$**um**ulations, $\mathbf{W}$-$\underline{\mathbf{C}}$**um**, onto $\mathbf{W}$ *isomorphically*:

$$\log_p\underline{\mathrm{Def}}[\,\underline{\partial}(\underline{C}_k \times \underline{C}_m)\,] = \log_p\underline{\mathrm{Def}}[\,\underline{\partial}(\underline{C}_{k+m})\,] = \log_p\underline{\mathrm{Def}}(\underline{q}_{k+m}) = k + m$$

$$= \log_p\underline{\mathrm{Def}}(\underline{q}_k) + \log_p\underline{\mathrm{Def}}(\underline{q}_m) = \log_p\underline{\mathrm{Def}}(\underline{\partial C}_k) + \log_p\underline{\mathrm{Def}}(\underline{\partial C}_m),$$

so,

$$\log_p\underline{\mathrm{Def}\partial}(\underline{C}_k \times \underline{C}_m) = \log_p\underline{\mathrm{Def}\partial}(\underline{C}_k) + \log_p\underline{\mathrm{Def}\partial}(\underline{C}_m).$$

Thus, $\log_p \underline{Def}(\ )$ or $\log_p \underline{Def\partial}(\ )$ offer us interesting alternatives as quantitative measures of **q**ualitative definiteness.

A "full circle" relationship of functional composition: $\underline{Def}(\ )$ and $\log_p\underline{Def}(\ )$.
One cannot help but notice that all of these compositions of functions, taken together, interconnect to form a "full circle" relationship:

$$k = \log_p\underline{Def}(\underline{q}_k) = \log_p\underline{Def}(\underline{\partial C}_k) = \log_p\underline{Def\partial}(\underline{C}_k) = \log_p\underline{Def\partial}[\,exQ(k)\,] = \log_p\underline{Def\partial}exQ(k) = id_{\mathbf{N}}(k),$$

or,

$$\log_p \underline{Def}\,\underline{\partial}exQ(\ ) = id_{\mathbf{N}}(\ ),$$ the identity element (on **N**) for the "composition of functions" operation,

$$\Rightarrow \log_p\underline{Def}(\ ) = [\,\underline{\partial}exQ\,]^{-1}(\ ) = [\,exQ\,]^{-1}[\,\underline{\partial}\,]^{-1}(\ ) = \underline{loQ\int}(\ )$$

$$\Rightarrow \log_p\underline{Def}(\ ) = \underline{loQ\int}(\ )$$

(such that '=' as used above denotes function[al] "equivalence" or "identity").

This last equation is an "identity" (equivalence) by definitions of the functions, each function acting on $_{\mathbf{w}}\underline{\mathbf{Q}}$ space elements. It expresses two "views" (similar to the way in which Maxwell's electromagnetic field equations do, as discussed by Thomas K. Simpson in his new book, ***Newton, Maxwell, Marx***):

View **1**: $\log_p\underline{Def}(\underline{q}_k) = \log_p(p^k) = k$; this mapping is *from* $_{\mathbf{w}}\underline{\mathbf{Q}}$ *through* $\{p^k\}$ *onto* **W**;

View **2**: $\underline{loQ\int}(\underline{q}_k) = loQ(\underline{C}_k) = k$; this mapping is *from* $_{\mathbf{w}}\underline{\mathbf{Q}}$ *through* $\underline{\mathbf{C}}_{\mathbf{w}}$ *onto* **W**.

Possibly an *"Astonishing Result"*

We have saved for last what may be *our most astonishing finding*. We must now interpret '$\underline{Def}(q_0) = p^0 = 1$' as asserting that the "null ontology", q_0, has exactly *one* "logical **e**lement"! But, wait a minute! Isn't q_0 supposed to be like the null set, { }, which has n**0** elements? (Note that $\log_p\underline{Def}(q_0) = 0$, however.) Rather than hastily "re-defining" the generalized "order function", on/at q_0, to be '$\underline{Def}(q_0) := 0$', let's consider simply *accepting* its implication:

■ *The "set-number"* q_0 *has* **1** *logical element, perhaps one which is somehow "uncounted" and "unseen"!*

This would make sense if we regarded --

$$q_0 := q_0 + \{\underline{C}_1\}^0 = q_0 + \text{'}\underline{C}_1\ \underline{un}\text{-crossed with itself'} = q_0 + \text{'}\{\underline{C}_1\{/\}\underline{C}_1\}\text{'} = q_0 + \{q_0\} = q_0$$

-- **a set containing itself !?**

This is the enigma we are left with: *That the origin-***q***ualifier "set-number" may have one "unseen logical element" within it – itself!* And this makes complete sense if q_0 is to "contain" the "essential idea" or "topic" of the entire assigned ontology, of which it is the "origin". ■

-- Joy-to-You !

F.E.D.** = **Foundation *Encyclopedia Dialectica, authors of the book:

A Dialectical "Theory of Everything" –

***Meta-Genealogies of the Universe and of Its Sub-Universes*:**

A Graphical Manifesto, **Volume 0**: **Foundations**.

Websites providing free download of **F.*E*.*D*.** "primer" texts include --

www.dialectics.org and **www.adventures-in-dialectics.org**