Series: Two Key ‘***Ideo***-Intra-Multi-alities’.

Part **1**.: ‘***Ideo***-Intra-Multi-ality’ and Self-Expanding [Hyper-]Dimensionality.

Series Introduction. Per the Seldonian theory, it is ‘intra-duality’ or ‘self-duality’ [more generally, ‘intra-***multi***ality’ or ‘self-***multi***ality’] -- i.e., ‘self-antithesis’, or ‘internal antithesis’ -- that is the cause of dialectic.

For examples of ‘***physio***-intra-duality’ or of ‘***physio***-self-duality’ -- of internal duality in objects that we perceive, with our minds, as residing outside of our minds -- we need only look to the most populous objects that we can see, with our naked eyes, in our skies: the stars.

Each star is a ‘‘‘complex unity’’’ of an ongoing, colossal, thermonuclear ***EX***plosion together with an ongoing, gargantuan, self-gravitational ***IM***plosion, locked in mutual modulation.

If something more “one-sided” were afoot with and within stars -- e.g., as we can imagine, if only an ***EX***plosion “side” ‘monality’ of their actual inner duality pertained, or if only an ***IM***plosion “side” ‘monality’ of their actual inner duality pertained, stars would manifest for but microseconds before self-destructing -- into the “absolute” implosion that forms “black-hole” ‘holonium’, or into the “absolute” explosion that would see a total expulsion of their mass.

Instead, typical stars last long -- for billions of years -- because of the mutually-checking [self-] interaction of these two -- opposite -- forces, *both* arising from the same [it]self. This interaction of internal opposites, i.e., of ‘self-opposites’, ineluctably co-present in that single [it]self that is a star, drives “stellar evolution”, and ‘stellar ***meta***-finite ***meta***-evolution’, through all of their stages, for each such individual star.

But, in the end, it is also this ***physical*** ‘intra-duality’ self-interaction, or ‘intra-action’, of each such star, as a whole, that brings the ‘‘‘life’’’ of each such star to its dramatic, combined, expulsive/implosive end, its outer layers massively ejecting, while its inner core becomes massively self-contracted and hyper-compressed.

There still remains the prickly matter of ‘***ideo***-intra-duality’; of the ‘intra-duality’ of the ‘‘‘thought-forms’’’ or of the ‘‘‘thought-contents’’’ -- of the ‘‘‘thought-objects’’’ -- that we humanoids uniquely create, and that are human ***mental*** objects ***only***; that have no [exact] counterparts in human ‘‘‘external’’’, physical experience. Plus of the systematic-dialectical, method-of-presentation dialectic that such ‘***ideo***-intra-duality’ may enable.

¿Does ‘***ideo***-intra-duality’ really exist***?***

¿If so, does it, in general, exhibit the same generic ‘‘‘dynamical’’’ and ‘meta-dynamical’ ‘‘‘laws-of-motion’’’ [patterns] that ‘***physio***-intra-duality’ does***?***

¿And, if it does, ***why*** does it***?***

Let us approach these questions by scrutinizing two of the paradigmatic cases of ***E.D.*** ‘‘‘systematic dialectic’’’, namely --

(**1**) The case of the presentational-dialectical systems-progression/categorial-progression up to the “***first***-order” Seldonian ‘First Arithmetic for Dialectic’, **N** –∋ **N** ⊢⊕⊣ $_{\mathbf{N}}$**Q**,

and;

(**2**) The case of the presentational dialectic of the [axioms-]systems/categorial-progression of the “Standard” Arithmetics of “***second***-order”, **N** –∋ **W** –∋ **Z** –∋ **Q** –∋ **R** –∋ **C**, and beyond.

Part 1. ‘***Ideo***-intra-duality’ in the Dialectical Presentation of the Axioms-Systems-Progression/-Categories-Progression, *from* the “Standard”, “First-Order”, “Natural” Numbers Arithmetic, *to* the Seldonian ‘First Arithmetic for Dialectic’, the latter being a “***Non***-Standard” “First-Order” Model of that “Standard” Arithmetic.

‘***Qualitative Ordinality***’. The conceptual, intuitive essence of the “**N**atural” Numbers of “***first***-order” is a specific kind within what **we** symbolize by ‘–∋’ -- connoting ‘***successionality***’, ‘***followership***’, “***sequentiality***”, ‘***seriality***’, ‘***consecutivity***’, or ‘***ordinality***’. About this first Part case, Seldon cites the core of an ‘***ideo***-intra-duality’ that conceptually energizes his dialectical presentation of his systems-progression of arithmetical/algebraical axioms-systems for modeling dialectic mathematically. He cites the inner intuitive ambiguity and internal tension, or ‘***in***-tension’, within the idea of ‘ordinality’, between standard ‘‘‘quantitative ordinality’’’ -- such as that of the familiar “ordinal” ‘‘‘numberings’’’, e.g., “***first***”, “***second***”, “***third***”, “***fourth***”, etc. -- and what he calls ‘qualitative ordinality’. By the latter, he means the quality shared in common, by all ***first*** categories, as such, in any known dialectical categorial progression, plus [‘∪’] the quality shared by all such ***second*** categories, plus that shared by all such ***third*** categories, plus that shared by all such ***fourth*** categories, etc.; the qualities of categories, in dialectical categorial progressions, of ‘***first-ness***’, of ‘***second-ness***’, of ‘***third-ness***’, of ‘***fourth-ness***’, etc.

The ‘‘‘***in***tension’’’ of, e.g., the ‘ordinal quality’ of ‘***second-ness***’ in a dialectical categorial progression can be modeled set-theoretically, i.e., by an “***ex***tension”, by the set of all categories from all known dialectical categorial progressions that constitute the ***second*** category in their native progressions. Per either the ‘Dyadic Seldon Function’ or the ‘Triadic Seldon Function’ generic interpretation, this quality is that of the ‘first *full* contra-category’ in dialectical categorial progressions in general. In the $_{\mathbf{N}}$**Q**, ‘Seldonian First System of Arithmetic/Algebra for Dialectic’, **we** express the intension of this set/extension by the symbol/‘meta-numeral’ -- ‘q_{2}’.

The '''***in***tension''' of the 'ordinal quality' of '***third-ness***' in a dialectical categorial progression can be modeled set-theoretically -- i.e., by an "***ex***tension" -- by the set of all categories from all known dialectical categorial progressions that constitute the ***third*** category in their native progressions. Per the generic interpretation native to either [both] the 'Dyadic Seldon Function' or [and] the 'Triadic Seldon Function', this quality is that of the 'first *full* uni-category' in dialectical categorial progressions in general. In the ${}_{\mathbf{N}}\mathbf{Q}$, 'Seldonian First System of Arithmetic for Dialectic', **we** express the intension of this set/extension by the 'idea-gram'/symbol 'ꟾ_{3}'.

The '''***in***tension''' of the 'ordinal quality' of '***first-ness***' in a dialectical categorial progression can be modeled, set-theoretically -- i.e., by an "***ex***tension" -- by the set of all categories from all known dialectical categorial progressions that constitute the ***initial and initiating*** category in their native progressions, the categories that have no predecessor category in their native categorial progressions. Per the generic interpretation native to either [both] the 'Dyadic Seldon Function' or [and] the 'Triadic Seldon Function', this quality is that of the '«***arché***»-category' in dialectical categorial progressions in general. In the ${}_{\mathbf{N}}\mathbf{Q}$, 'Seldonian First System of Arithmetic for Dialectic', **we** express the intension of this set/extension by the 'idea-gram'/symbol 'ꟾ_{1}'.

When **we** come to the '''***in***tension''' of the 'ordinal quality' of '***fourth-ness***' in a dialectical categorial progression, **we** find that, again, ***it*** can be modeled set-theoretically, by an "***ex***tension" -- by the set of all categories from all known dialectical categorial progressions that constitute the ***fourth*** category in their native progressions. But in this case, the generic interpretations of the 'Dyadic Seldon Function' and the 'Triadic Seldon Function' diverge. From the perspective of the 'Dyadic Seldon Function', this quality is that of the 'second *full* contra-category'. From the perspective of the 'Triadic Seldon Function', this quality is that of the 'first *partial* contra-category'. In the ${}_{\mathbf{N}}\mathbf{Q}$, 'Seldonian First System of Arithmetic for Dialectic', **we** express the intension of this set/extension by the "symbol", or 'meta-numeral' ideogram, 'ꟾ_{4}'.

The quality of 'qualitative ordinality in general' can then be expressed, generically, by the set of all sets representing individual, specific ordinal qualities, including the four specific ordinal qualities described above. This set-model of 'qualitative ordinality' in general is a ***finite*** set, whose highest ordinal set-element -- say its **k**th ordinal set-element, such that **k >>> 2** -- is the set of all categories from all known dialectical categorial progressions that constitute the **k**th category in their native dialectical categorial progressions. This means that there is no then-known dialectical categorial progression that has more than **k** categories actualized in it.

Thus, when **we** '''assign''' or interpret ['$\in\!\!\dashv$'] a category [of axioms-systems] like **N**, attributing the category of the first-order "Standard Model" axioms-systems for the "**N**atural" Numbers to the 'meta-numeral' ꟾ_{1} of the ${}_{\mathbf{N}}\mathbf{Q}$ set of 'meta-numerals', by signing '$\text{ꟾ}_{1} \in\!\!\dashv \mathbf{N}$', what we are asserting, in set-theoretical interpretation, is that **N** is an element of the set of all first categories of all known dialectical categorial progressions, or, in ideographical symbols: $\mathbf{N} \in \text{set}(\text{ꟾ}_{1})$ --

$$[\mathbf{N} \vdash\!\!\ni \text{ꟾ}_{1}] \leftrightarrow [\mathbf{N} \in \text{extension}(\text{ꟾ}_{1})].$$

And, when we '''assign''' or interpret ['⊂—∃'] a category [e.g., of axioms-systems] like $_{N}Q$, attributing this category, of ***first***-order "***Non***-Standard Model" axioms-systems, that for the '***Meta***-Natural' Numbers, to the 'meta-numeral' $ɿ_2$ of the $_{N}Q$ ***non***-standard set, or "space", of such 'meta-numerals', by means of the expression '$ɿ_2$ ⊂—∃ $_{N}Q$', what we are asserting, in set-theoretical interpretation, is that $_{N}Q$ is an element of the set of all ***second*** categories of all known such categorial progressions, or: $_{N}Q$ ∈ set($ɿ_2$) -- [$_{N}Q$ ⊢∋ $ɿ_2$] ⊂∋ [$_{N}Q$ ∈ extension($ɿ_2$)].

The concept of 'ordinality' forms the "***First***-Order" core intuitive meaning of the "Standard Model" of the Peano-Dedekind axioms-system for "**N**atural" Numbers Arithmetic, the system which we name **N**. It does so via the first four, "***first***-order-logic" level axioms, the first four "Peano-Dedekind Postulates", formulated within "***first***-order logic", whereas the fifth of those "Dedekind-Peano Postulates" is formulated within "***second***-order logic". The first four, "***first***-order" axioms are, if we, '''standardly''', use **N** to denote the set of all "**N**atural" numbers --

1. ⊩ '''**1** is a "**N**atural" number.''', or '''[**1** ∈ **N**]'''.

2. ⊩ '''If ς is a "**N**atural" number, then the **s**uccessor of ς is also a "**N**atural" number.''', or '''[[ς ∈ **N**] ⇒ [s(ς) ∈ **N**]]'''. [Note: [∀n ∈ **N**][s(n) = n + **1**]].

3. ⊩ '''If **n** & **m** are "**N**atural" numbers, & if **n** differs from **m**, then the **s**uccessor of **n** differs from the **s**uccessor of **m**.''' or '''[[[n, m ∈ **N**] & [n ≠ m]] ⇒ [s(n) ≠ s(m)]]'''.

4. ⊩ '''There is no "**N**atural" number *such that* ['|'] **1** is the **s**uccessor of that number ['**1** has no predecessor in **N**.', or 'The "**N**atural" number **1** exhibits the characteristic of lacking any "**N**atural" number predecessor.'].''', or '''[[¬∃x ∈ **N**] | [s(x) = **1**]]'''.

These axioms are *asserted* ['⊩'] *without proof*, i.e., they are ***assumed*** to be true, as premises. The subtle, inner, intuitive contrast between the '''quantitative ordinality''' species and the 'qualitative ordinality' species of the 'ordinality' «genos» -- the '***ideo***-intra-duality' of **N**, i.e., of the "Standard", "***first***-order", "**N**atural" Numbers [axioms-]system of arithmetic's ***idea*** of 'ordinality-in-general' -- is part of Seldon's intuitive explanation for the implied, "logically necessary" ***co***-existence of "***Non***-Standard Models" of "***first***-order **N**atural" Numbers arithmetic, existent together with the "Standard Model", "**N**atural" arithmetic, once the logical "existence" of the "Standard Model of the **N**atural Numbers" arithmetic is posited. The immanent ***critique***, or *self*-***critique*** -- **N**(**N**) ≡ ↩(**N**), [wherein '↩≡ **N**', *for* **N**] the dialectical, determinate [***self***-]***negation*** ['↩'] of **N** -- i.e., of the standard ***first***-order system, **N**, plus ***our*** '''Lakatosian counter-example system''' solution ['|-≡'] for the resulting ***second*** term, as expressed in the $_{N}Q$ arithmetical/algebraic language itself --

N —∋ **NN**, ≡ **N**(**N**) ≡ **N**⊗**N** ≡ $\mathbf{N}^2$ ≡ ↩**N** ≡ **N** ⊕ ***Delta*****N** ≡ **N** ⊕ $ɿ_{NN}$ |-≡ **N** ⊢⊕⊣ $_{N}Q$

-- is driven, motivated, and energized by the emerging evocation of that '***ideo***-intra-duality'.

***The* “*Orders*” *of the Formal-Logical Ideographical Languages* [**i.e., *of the* “*Predicate Calculi*”].**
“***First***-order” “**N**atural” Numbers logic makes direct assertions -- as can be seen in the first four, “***first***-order” Peano-Dedekind axioms for **N**, stated above -- only about the characteristics of those ‘‘‘***idea***-objects’’’ that are single, ***individual*** “**N**atural” Numbers.

That “***first***-order” logic language makes no direct assertions about those ‘‘‘***idea***-objects’’’ that are the qualities shared in common by whole groups of “**N**atural” Numbers, i.e., extensionally, by sets that have two or more “**N**atural Numbers” as their ‘ultimate elements’.

Examples include the ‘‘‘***idea***-object’’’ that is the quality shared in common by all of the numbers in that ‘‘‘sub-group’’’ of the “**N**atural” Numbers called the “***odd*** [“**N**atural”] Numbers” [vs. that of the quality shared in common by all of the so-called “***even*** [“**N**atural”] Numbers”], or the ‘‘‘***idea***-object’’’ that is the quality shared in common by all numbers in that ‘‘‘sub-group’’’ of the “**N**atural” Numbers called the “***prime*** [“**N**atural”] Numbers” [vs. that of the quality shared in common by all of the so-called “***composite*** [“**N**atural”] Numbers”].

Direct assertions about those ‘‘‘***idea***-objects’’’ that are the [meta^{0}-]qualities shared by entire groups, or sets, of two or more “**N**atural” Numbers, belong to the language of “***second***-order predicate logic”.

“***First***-order” “**N**atural” Numbers logic language also makes no direct assertions regarding those ‘‘‘***idea***-objects’’’ which are the ‘meta^{1}-qualities’, or ‘qualities of qualities’ [‘predicates of predicates’], shared in common by ‘groups of groups’ -- or by “sets of sets” -- that have “**N**atural” **N**umbers as their ‘ultimate elements’.

Such assertions belong to the language of “***third***-order predicate logic”.

An example of a “**3*rd***-order” predicate that is sometimes given is that of the quality that can be named ‘‘‘***Discontinuousness***’’’, as evoked in the assertion “There is a ***discontinuous*** function from **R** to **R**.”, or ‘ $\vdash [\exists f \in F] | [[{}^{3}D^{1}(f)] \;\&\; [f: \mathbf{R} \rightarrow \mathbf{R}]]$’, wherein **R** denotes the set of all “**R**eal” Numbers, as contained in the “**2*nd***-order” axioms-system that **we** denote by **R**. If one holds that this **f**unction, call it ‘$f: \mathbf{R} \rightarrow \mathbf{R}$’, is already an ‘*idea*-object’ of “**2*nd***-order”, in [‘$\in$’] the set of *F*unctions, *F*, & that ‘‘‘***Discontinuousness***’’’ [e.g., as symbolized, within a “**3*rd***-order Predicate Calculus”, by, e.g., ‘${}^{3}D^{1}(\_)$’] is a predicate applicable to **R**eal-valued **f**unctions, expressible ideographically by ‘ $\vdash {}^{3}D^{1}(f)$’, with “truth-value” ‘TRUE’ asserted [‘ $\vdash$ ’] -- then ‘${}^{3}D^{1}(\_)$’ would represent a **1**-place [‘$(\_)$’] predicate of “**3*rd***-order”.

“***First***-order” “**N**atural” Numbers logic language also makes no assertions regarding those ‘‘‘***idea***-objects’’’ which are the ‘meta^{2}-qualities’, i.e., ‘qualities of qualities of qualities’, shared in common by groups of ‘meta^{1}-qualities’, i.e., of ‘qualities of qualities’ -- i.e., by “sets of sets of sets” -- ultimately having “**N**atural” Numbers as their ‘ultimate elements’. Such assertions belong to the language of “***fourth***-order predicate logic”, whose extension is the set of all functions that share the quality of ‘‘‘***Discontinuousness***’’’. Note also here another instance of the pattern, of ‘dialecticality’, i.e., of ‘qualo-fractal’, «***aufheben***» ‘meta-unit-ization’, as the iterated ‘meta-***element***-izations’ of sets: ***elevation***, ***conservation***, & ***determinate negation*** of former “ultimate” sets, as units, into mere elements of ‘meta-sets’, grasped as ‘[meta-]units’.

"***Fourth***-order predicates", representing 'meta[1]-meta[1]-qualities', or 'meta[2]-qualities', i.e., 'qualities of qualities of qualities' ['predicates of predicates of predicates'], apply to sets of multiple "***third***-order predicates" -- sets whose immediate elements are the sets/extensions of "***third***-order predicates" -- i.e., apply to qualities that groups of "***third***-order" predicates share.

"***Third***-order predicates", representing 'meta-qualities', or 'meta[1]-qualities', i.e., 'qualities of qualities' ['predicates of predicates'], apply, extensionally, to sets of multiple "***second***-order predicates" -- sets whose immediate elements are ***second***-order predicates -- i.e., apply to qualities that groups of "***second***-order" predicates share in common.

"***Second***-order predicates", representing qualities shared in common by two or more individual "**N**atural" Numbers, apply to sets of two or more "**N**atural" Numbers, sets whose immediate elements are also the 'ultimate elements' in this ***Domain*** -- individual "**N**atural" Numbers -- i.e., apply to qualities that groups of such numbers share in common.

Thus, a single "***fourth***-order predicate", as a 'set-unit', is, extensionally, an «***aufheben***» 'meta-element-ization' of multiple "***third***-order predicates" -- i.e., is made up out of the heterogeneous multiplicity of its sub-units -- that are the extensions of "***third***-order predicates" -- as its immediate set elements.

A single "***third***-order predicate", as a 'set-unit', is, extensionally, an «***aufheben***» 'meta-element-ization' of multiple "***second***-order predicates" -- i.e., is made up out of the heterogeneous multiplicity of its sub-units -- that are the extensions of "***second***-order predicates" -- as its immediate set elements.

A single "***second***-order predicate", taken as an extensional unit, or 'set-unit', is an «***aufheben***» 'meta-element-ization' of multiple "***first***-order predicates" -- which are made up out of the multiplicity of those "logical individuals" that satisfy "***first***-order predicates" as their immediate set elements, which are also the 'ultimate set elements' here -- individual "**N**atural" Numbers.

A "***first***-order predicate" is a predicate that applies to ***single***, ***individual*** "**N**atural" Numbers only, one at a time, and ***not***, all at once, to groups -- sets -- of "**N**atural" Numbers.

A Formal-Logical, Partially-Constructive Anticipation of NON-Standard "Natural" Numbers. Formally, if not fully "constructively", the logical "existence" of "***non***-standard models" of the "***first***-order **N**atural" Numbers is implied by the "***first***-order" co-application of two of the deepest "meta-mathematical" theorems known. Both were originally proven by that magisterial 'logician of logicians', Kurt Gödel [1906–1978]. These two theorems "sound" mutually contradictory, but are actually profoundly revelatory in their "***first***-order" conjoint implications. They are the "Gödel Completeness Theorem" for "***first***-order logic" [only], and the "Gödel First Incompleteness Theorem", which addresses ***higher***-order logics" as well as "***first***-order logic".

At the level of "***first***-order logic" the completeness theorem applies "semantically", while the incompleteness theorem applies "syntactically" [for ***second***- and ***higher***-order logic, only the first incompleteness theorem applies [both "semantically" and "syntactically"]]. "Semantics", or the "meaning" dimension of predicate calculus formulas, in this formal-logical context, reduces to the **2** [toggling] meanings of "[deductively] True" and "[deductively] False" [We are ***not*** talking, here, about any kind of "absolute" truth, but only about truth *relative* to assumptions, i.e., *relative* to axioms, such that the axioms themselves are **unproven**, even if *supposedly* -- but often not *really* -- "self-evident"].

The "semantic" completeness of the "standard model" of the "**N**atural" arithmetic of "***first*** order" means that its "***first***-order logic" is sufficient to prove, deductively, within that arithmetic, every assertion of that arithmetic with the meaning "True" within that "standard" arithmetic.

The "syntactic" incompleteness of the "standard model" of the "**N**atural" arithmetic of "***first*** order" means that its "***first***-order logic" is insufficient to *either* prove *or* disproof -- to "decide" deductively, within that arithmetic -- every possible "well-formed" assertion of that arithmetic, as to its truth or falsity therein. This means that there are some "well-formed" [correctly formed '''grammatically'''] assertions within the "standard model" of the "**N**atural" arithmetic of "***first*** order", which are "undecidable" within the "***first***-order logic" language of that arithmetic -- i.e., which can *neither* be deductively proven *nor* deductively refuted within it.

For this to be true, it means that there must be one or more "well-formed" assertions within "***first***-order" arithmetic that are deductively true in the "standard model" of that arithmetic, but deductively false in at least one other "***first***-order" model of that same axioms-system of arithmetic -- that other model being, therefore, a "***non***-standard model" of the "***first***-order" Peano-Dedekind axioms of that [axioms-]*system of arithmetic*.

And/or, there must be one or more "well-formed" assertions within "***first***-order" arithmetic that are deductively false in the "standard model" of that arithmetic, but deductively true in at least one other "***first***-order" model of that system of arithmetic -- of its axioms -- that other model being therefore a "***non***-standard" model of the "***first***-order" Peano-Dedekind axioms.

My favorite rendition of this state of affairs was written by the mathematical logician John W. Dawson, Jr., in his biography of Kurt Gödel, as follows -- "Most discussions of Gödel's proof....focus on its quasi-paradoxical nature. It is illuminating, however, to ignore the proof and ponder the implications of the theorems themselves."

"It is particularly enlightening to consider together both the completeness and incompleteness theorems and to clarify the terminology, since the names of the two theorems might wrongly be taken to imply their incompatibility."

"The confusion arises from the two different senses in which the term "complete" is used within logic."

"In the semantic sense, "complete" means "capable of proving whatever is valid" whereas in the syntactic sense it means "capable of proving or refuting each [**F**.***E***.***D***.: "well-formed"] sentence of the theory." [**F**.***E***.***D***.: Thus, for *syntactic* completeness, there must be ***no*** "well-formed" sentence of **N** which **N**'s axioms can ***neither*** prove ***nor*** dis-prove, deductively, i.e., there must be ***no*** "undecidable" such sentences.].

"Gödel's completeness theorem states that every ... first-order theory, whatever its nonlogical axioms may be, is complete in the former [semantic] sense: Its theorems coincide with statements true in *all* models of its axioms [**F**.***E***.***D***.: Thus, Dawson is referencing the idea that there can be more than one model of a first-order theory, before just-yet actually explicating this idea]."

"The incompleteness theorems, on the other hand, show that if formal number theory is ***consistent*** [**F**.***E***.***D***.: I.e., if the standard theory of the "Natural" numbers of "first order" is ***non***-self-contradictory*****], it fails to be complete in the second [syntactic] sense."

*[F.E.D.: *Note* -- A "Gödel Formula", call it **G**, as constructed in Gödel's "First Incompleteness Theorem", is a '''self-reflexive''', "self-referential" formula of the kind about which Russell so bitterly complained, as the cause of self-contradictory, nonlinear ["quadratic"] "propositional functions" afflicting his & Frege's formal-logical arithmetics when unguarded by Russell's types-hierarchy-theory. Gödel's "self-referential" formula is both ***non***-self-contradictory and only "quasi-paradoxical". A **G** asserts *of itself* 'I am not deductively provable from the axioms' of the arithmetic within which it is constructed. If that assertion is true, then that arithmetic is ***in***complete -- ***un***able to deductively derive one of its own, internally true, "well-formed" formulas/assertions. If **G** is false, then **G** ***is*** deductively derivable from the axioms of that arithmetic. That arithmetic is then therefore ***in***consistent, or self-contradictory -- able to deductively derive, from its axioms, an assertion/formula which is false within it. Thus, Gödel's first incompleteness theorem essentially asserts: '''If this arithmetic is consistent, then it is incomplete.''']

"The incompleteness theorems hold also for higher-order formalizations of number theory. If only first-order formalizations are considered, then the completeness theorem applies as well, and together they yield not a contradiction, but an interesting conclusion: Any sentence of arithmetic that is undecidable [F.E.D.: I.e., any assertion which can ***neither*** be ***proven*** nor ***refuted*** by deduction from the "first-order", **N**, axioms] must be ***true*** in ***some*** models of Peano's axioms (lest it be formally ***refutable*** [F.E.D.: As it would be if it were ***false*** in ***all*** models of the "first-order" Peano axioms]) and ***false*** in [F.E.D.: ***some*** of the] others (lest it be formally ***provable*** [F.E.D.: As it would be if it were ***true*** in ***all*** models of the "first-order" Peano-Dedekind Postulates]). In particular, there must *be* models of first-order Peano arithmetic whose ***elements*** [F.E.D.: I.e., whose "numbers" [and their "numerals"]] do not "behave" the same as the [F.E.D.: "Standard"] natural numbers."

"Such nonstandard models were ***unforeseen*** [F.E.D.: By ***non***-dialectical thinkers, who presume monolithic '***ideo***-intra-monality', instead of '***ideo***-intra-multiality', for ***all orders*** of formalized human '''***idea***-objects'''] and ***unintended***, but they cannot be ignored, for their existence implies that *no first-order axiomatization of number theory can be adequate to the task of deriving as theorems* **exactly** *those statements that are true of the* [F.E.D.: "Standard"] *natural numbers*."

[from John W. Dawson, Jr., ***Logical Dilemmas***: ***The Life and Work of Kurt Gödel***, Wellesley, MA, A K Peters: 1997, pp. 67–68, ***emphases*** added by F.E.D..].

One of those "***first***-order logic ***non***-standard models" of the Peano-Dedekind Postulates is the axioms-system of the Seldonian "First Arithmetic for Dialectic", which **we** denote by $_{\mathbf{N}}\mathbf{Q}$. $\mathbf{I^{t}s}$ 'meta-numbers' abide by a version of the first four, "***first***-order" Peano-Dedekind axioms, but also differ dramatically from the "Standard **N**atural" numbers of the "Standard **N**atural Model", or **N**, arithmetic. More specifically, the axioms of $_{\mathbf{N}}\mathbf{Q}$ 'explicitize' an arithmetic that is interpretable as expressing "purely" 'qualitative ordinality', in contrast to the "purely" 'quantitative ordinality' expressed, outwardly, on its surface, by the "Standard Model" of the "**N**atural" Numbers arithmetic of "first order".

The inescapable internal '''ambiguity''' of the "***first***-order" axiomatic specification of "**N**atural" Arithmetic, its ineluctable '***ideo***-intra-duality' -- or '***ideo***-intra-multiality', as there are more than two possible models of "***first***-order" Peano-Dedekind arithmetic -- inherently and immanently "leaves room" for alterative models of that arithmetic, and even for a model which is an ***extreme*** '***supplementary opposite***' of the standard model, namely for the $_{\mathbf{N}}\mathbf{Q}$ model.

If the "*first*-order <u>standard</u> model **N**atural Arithmetic" can be interpreted as a vocabulary of numbers which are '<u>un</u>qualified ordinal arithmetical <u>quant</u>ifiers', then the '***Non***-Standard ***Meta***-**N**atural Arithmetic' of the extreme-***opposite***, $_{\mathrm{N}}$**Q**, ***non***-standard model can be interpreted as a vocabulary of '<u>meta</u>-numbers' which are '<u>un</u>quantifi<u>able</u> ordinal, arithmetical <u>ontological</u> <u>qual</u>ifiers', forming a set or "space" within the $_{\mathrm{N}}$**Q** axiom<u>s</u>-system which **we** name $_{\mathrm{N}}$**Q**. **Our** *core* axioms for the 'Seldonian First Arithmetic for Dialectic', $_{\mathrm{N}}$**Q** -- not *all* of $_{\mathrm{N}}$**Q**'s axioms, but $_{\mathrm{I}}$**t**$_{\mathrm{S}}$ *main* axioms -- which can be so interpreted, are the following --

§**1**. **The Axiom of the Beginning**. ■ '⟟$_{1}$ is a '***Meta***-**N**atural meta-number.', or -- '**[**⟟$_{1}$ ∈ $_{\mathrm{N}}$**Q]**';

§**2**. **The Axiom of the Succession**. ■ 'If ⟟$_{\varsigma}$ is a '***Meta***-**N**atural meta-number', then the **s**uccessor of ⟟$_{\varsigma}$ is also a '***Meta***-**N**atural meta-number.', or -- '**[[**⟟$_{\varsigma}$ ∈ $_{\mathrm{N}}$**Q] ⇒ [s(**⟟$_{\varsigma}$**)** ∈ $_{\mathrm{N}}$**Q]]**';

§**3**. **The Axiom of 'Successorial' Uniqueness**. ■ 'If ⟟$_{n}$ **&** ⟟$_{m}$ are '***Meta***-**N**atural meta-number<u>s</u>', **&** if ⟟$_{n}$ differs from ⟟$_{m}$, then the **s**uccessor of ⟟$_{n}$ differs from the **s**uccessor of ⟟$_{m}$.', or -- '**[[∀**⟟$_{n}$, ⟟$_{m}$ ∈ $_{\mathrm{N}}$**Q][[**⟟$_{n}$ ≠ ⟟$_{m}$**] ⇒ [s(**⟟$_{n}$**) ≠ s(**⟟$_{m}$**)]]]**';

§**4**. **The Axiom of 'Archéonicity'**. ■ 'There is no '***Meta***-**N**atural meta-number' such that ⟟$_{1}$ is the **s**uccessor of that 'meta-number'.'['⟟$_{1}$ has no <u>pre</u>decessor in $_{\mathrm{N}}$**Q**.', i.e., 'The '***Meta***-**N**atural meta-number' ⟟$_{1}$ exhibits the characteristic of an «***arché***», i.e., of lacking any '***Meta***-**N**atural meta-number' <u>pre</u>decessor in $_{\mathrm{N}}$**Q**.'], or -- '**[[¬∃**⟟$_{x}$ ∈ $_{\mathrm{N}}$**Q] | [s(**⟟$_{x}$**) =** ⟟$_{1}$**]]**';

§**5**. **The Axiom of Interconnection**. ■ 'For every "*first*-order <u>standard</u> **N**atural" number, the '***Meta***-**N**atural meta-number' with that <u>standard</u> number as its subscript/denomination is an element of the set of all such '***Meta***-numbers', named $_{\mathrm{N}}$**Q**.', or -- '**[[∀n ∈ N][**⟟$_{n}$ ∈ $_{\mathrm{N}}$**Q]]**';

§**6**. **The Axiom of Qualitative Inequality**. ■ 'For all '***Meta***-**N**atural meta-numbers', if their <u>subscripts</u> are <u>un</u>equal <u>quant</u>itatively [' ⋛ '], then <u>they</u> <u>un</u>equal <u>qual</u>itatively [' ⧧ '].', or -- '**[[∀**⟟$_{j}$, ⟟$_{k}$ ∈ $_{\mathrm{N}}$**Q][[j ⋛ k] ⇔ [**⟟$_{j}$ ⧧ ⟟$_{k}$**]]]**';

§**7**. **The Axiom of the Idempotency of Self-Addition**. ■ 'The sum of **2** "copies" of a single '***Meta***-**N**atural meta-number' equals a single "copy" thereof.', or -- '**[[∀**⟟$_{n}$ ∈ $_{\mathrm{N}}$**Q][** ⟟$_{n}$ ⊞ ⟟$_{n}$ **=** ⟟$_{n}$**]]**';

§**8**. **The Axiom of Self-Transcendence**. ■ 'There is no '***Meta***-**N**atural meta-number' equal to the sum of <u>*distinct*</u> such 'meta-numbers'.' -- '**[[∀**⟟$_{n}$, ⟟$_{j}$, ⟟$_{k}$ ∈ $_{\mathrm{N}}$**Q][[**⟟$_{j}$ ≠ ⟟$_{k}$**]⇒[**⟟$_{j}$ ⊞ ⟟$_{k}$ ≠ ⟟$_{n}$ **]]]**';
[This axiom implies the '''non-closure''', or '''open-ness''', of the set $_{\mathrm{N}}$**Q** to its own, $_{\mathrm{N}}$**Q**, '''addition''' operation.].

§9. **The Axiom of Production**. ⊩ 'The product ['⊠'] of any two '***Meta*-N**atural meta-numbers', distinct or not, equals the multiplicand 'meta-number' plus ['⊞'] that 'meta-number' whose subscript is the sum of the subscripts of the multiplicand 'meta-number' and the multiplier 'meta-number'.', or -- '[[∀ⵑ$_j$, ⵑ$_k$ ∈ $_N$**Q**][ⵑ$_j$ ⊠ ⵑ$_k$ = ⵑ$_k$ ⊞ ⵑ$_{k+j}$]]'. [This axiom implies the '''open-ness''' of the set $_N$**Q** to its own, $_N$**Q**, '''multiplication''' operation, via its additive '''open-ness'''.].

To better grasp the axiomatic connection between $_N$**Q** and **N**, it may be helpful to consider a '''generic''' or 'genericized' version of the "***first***-order" Peano-Dedekind axioms, collectively named **X**, addressing a generic "first-*in-order* number", call it **a**, in a generic number-set, or number-space within **X**, call it **X**, with a 'genericized' **s**uccessor function, call it **s** --

§α. ⊩ '**a** ∈ **X**';

§β. ⊩ '[[**x** ∈ **X**] ⇒ [**s**(**x**) ∈ **X**]]';

§γ. ⊩ '[[[**b**, **c** ∈ **X**] & [**b** ≠ **c**]] ⇒ [**s**(**b**) ≠ **s**(**c**)]]';

§δ. ⊩ '[[¬∃**x** ∈ **X**] | [**s**(**x**) = **a**]]'.

The above, 'genericized' version of the first four, "***first***-order" Peano-Dedekind Postulates "cover", as «genos», or implicitly & immanently "contain", both of the '''species''' of those axioms addressed herein, namely, the "standard", **N**, '''species''' & the '''***non***-standard''', $_N$**Q** '''species'''. The **s**uccessor functions that inhere in **N** & in $_N$**Q**, herein, named **s** and **s**, respectively, are, again, special cases of the 'genericized' **s**uccessor function, invoked just above, namely, **s**, such that **s** «***aufheben***»-'''contains''' ['⊏','⊐'] both, in a systematic, «***Genos***»/«***species***» sense: **s** ⊏ **s** ⊐ **s**. But, because $_N$**Q** is ***not*** a '''radical dual''' of **N**, ***not part*** of some "absolute" ['''Manichean'''] dualism between **N** & $_N$**Q**, **s** & **s** are intimately interconnected. Because $_N$**Q** is an «***aufheben***» ***opposite*** ['⮐'], i.e., a dialectical ***opposite***, or a '***supplementary opposite***' of **N** ['⊢. $_N$**Q** ⮐ **N**'] -- i.e., because $_N$**Q** is an ***opposite*** of **N** that nevertheless pre-supposes **N**, that «***aufheben***»-***conserves*** & «***aufheben***»-***elevates*** as well as «***aufheben***»-***determinately negates*** **N**, $_N$**Q**'s **s** is defined & built from/upon **N**'s **s**. I.e., for all **n** in **N** -- **s**[ⵑ$_n$] ≡ ⵑ$_{s(n)}$ = ⵑ$_{n+1}$. $_N$**Q** is an ***opposite*** of **N** that is born out of **N** itself, that is '***ideo***-meta-genealogically' related to & given-birth-to by **N** itself -- i.e., when **N** is 'mentally-embodied' by human agents -- as a result of **N**'s '***ideo***-intra-duality'. [***Note***: The Gödelian "First Incompleteness Theorem" & "Completeness Theorem" are not entirely "non-constructive" in their implication of the logical co-existence of ***non***-standard models of **N** given the logical existence of the standard model of **N**. The "First Incompleteness Theorem" constructs an assertion, call it **G**, which is true in **N**, but ***not*** deductively decidable from **N**'s axioms [& which "deformalizes" to a "diophantine" [algebraic] equation unsolvable in **N**, but not provably so within **N**]. This implies that **G** must be false within *some* model(s) of **N**. Satisfying ¬**G** is a *sufficient condition*, though ***not*** a *necessary condition*, for a model of **N** to be a ***non***-standard model of **N**. There are also ***non***-standard models of **N** within which **G** is true.].

Analytical-Geometric Visualization of the 'Ideo-Ontological Change' ['△'] from N to N ⊢⊕⊣ $_{N}Q$.

A key observation to note, to help discern the qualitative difference, the difference in ***kind***, the 'ideative', '***ideo***-ontological' difference, between the $_{N}Q$ arithmetic and the **N** arithmetic, is this --

- Within the **N** universe[-of-discourse], the "standard" Peano **s**uccessor operator, **s** ⊏ **s**, moves from one "Standard **N**atural" Number to the next one, by, in '"standard"' depiction, jumping one unit length step to the right each time the **s** operation is applied. Iterated application of the **s** operation, starting with its application to the operand **1**, expresses, per this "**N**atural" Numbers '''analytical-geometrical''' picture; generation of a growing portion of the "**N**atural" '''number-line''' "space", **N**, piecewise, by a cumulative progression of 'unilineal' unit-length steps or jumps, all of them within a ***single***, monolithic direction/***dimension***, viz. [All "standard", **N**, numbers populate the same, single dimension]:

........• -- i.e., **1**;

........•........• = **s(........•)** -- i.e., $\mathbf{s(1) = 2}$;

........•........•........• = **s(........•........•)** -- i.e., $\mathbf{s(2) = s(s(1)) = 3}$;

........•........•........•........• = **s(........•........•........•)** -- i.e., $\mathbf{s(3) = s(s(s(1)) = s^{3}(1) = 4}$; ...

- In the $_{N}Q$ universe[-of-discourse], the '''***non***-standard''' Peano **s**uccessor operator, **s** ⊏ **s**, jumps from one *oriented* unit-length line segment to a *differently-oriented* unit-length line segment -- to a *perpendicular* next unit-direction or unit-***dimension***, one which is *mutually perpendicular* vis-a-vis every predecessor unit-***dimension*** within $_{N}Q$ -- each time the **s** operation is applied to such a unit direction/***dimension***, viz., generating, step-by-step, that progression of mutually-orthogonal unit-***dimensions*** that constitutes a finitely-growing portion of the finite, $_{N}Q$-inherent, '⩚-dimensional*****', 'meta-number' "[hyper-]space" that **we** name $_{N}Q$ [Each "non-standard" 'meta-number' occupies a different dimension -- its own unique dimension]:

$_{N}Q$ ≡ { q_1, q_2, q_3,..., $q_{⩚}$ }, viz. --

— .... ⊢⊣ q_1;

s(—) = | .... ⊢⊣ q_2;

s(|) = s(s(—)) = / .... ⊢⊣ q_3; ... .

Thus --

$_{N}Q$ ⊢⊣ { —, | , /,...} ⊢⊣ { q_1, q_2, q_3,...}, & q_1 ⊥ q_2 ⊥ q_3... ⊢⊣ q_1 ⫲ q_2 ⫲ q_3...

Note: These unit line-segment geometric representations of the $_{N}Q$ 'meta-numbers' are not "composed" of "**0**-dimensional", "infinitesimal", "points". From the limited perspective of the $_{N}Q$ axioms, they are ***solid*** line-segments, 'impartible', '''uncuttable''' -- '***a***-tom-ic'. Only as the human-mind-'''*embodied*''' & 'human-mind-*agented*' $_{N}Q$ system explores 'its' own internal incompletenesses, do higher, richer -- if still "non-standard" -- seeings, & higher, richer "non-standard" arithmetics for dialectics, come into view.

*****[The variable '⩚' denotes the highest "**N**atural" Number representable in the computer in present use.].

A Seldon Function, applied to q_1 as its ultimate, «***arché***»-argument, i.e., by enacting a self-iterating ***self-involution*** of that argument, will generate a series, as "***non***-amalgamative", «***asumbletoi***», vector-like '''sums''' of those successive unit-***dimensions***; *a progression of* ***spaces*** -- i.e., the [hypo-/hyper-] ***diagonals*** of a unit-space of ever-growing, ever-higher ***dimension***ality -- a progression of '[hypo-/hyper-] diagonals' of that '''unit-[hypo-/hyper-]cube's''' [hypo-/hyper-] cubical space. Thereby, each unit-direction/-***dimension*** can be interpreted as representing an ontological category, one that is qualitatively, ***ontologically*** not-equal-to ['≠', ∈∋ '⊥'] all the other such ***dimensions***/ontological categories then extant. Thus, as a whole, this space of [self-]expanding dimensionality can be used to model a [self-]expanding ontology -- either (1) a [self-]expanding '***physio***-ontology', such as that of our cosmos as a whole, or (2) a [self-]expanding '***ideo***-ontology', or (3) both, combined -- e.g., as in a human economic system. This constitutes ***our*** account of the '***ideo***-ontological difference' that grounds ***our*** assertions ['⊦'] of the qualitative, '***ideo***-*ontological*' inequality between $_{\mathbf{N}}\mathbf{Q}$ & $\mathbf{N}$ -- '⊦. $_{\mathbf{N}}\mathbf{Q}$ ≠ $\mathbf{N}$' -- & of the dialectical opposition relation ['⮐'] that holds via the '***supplementary*** oppositeness' of $_{\mathbf{N}}\mathbf{Q}$ *vs.* $\mathbf{N}$ -- '⊦. $_{\mathbf{N}}\mathbf{Q}$ ⮐ $\mathbf{N}$'.

All of this is implied by the $_{\mathbf{N}}\mathbf{Q}$ *gen*eric equation --

$$\mathsf{q}_1 \longrightarrow \mathsf{q}_1\mathsf{q}_{1'} \equiv \mathsf{q}_1[\![\mathsf{q}_1]\!] \equiv \mathsf{q}_1 \boxtimes \mathsf{q}_1 \equiv \mathsf{q}_1^{\,2} \equiv \lrcorner\mathsf{q}_1 \equiv \mathsf{q}_1 \boxplus \Delta\mathsf{q}_1 \equiv \mathsf{q}_1 \boxplus \mathsf{q}_{1+1} = \mathsf{q}_1 \boxplus \mathsf{q}_2$$

-- when that *gen*eric equation, above, is interpreted for, & substituted by, the *speci*fic category/system named $\mathbf{N}$ in the equation below, given '$\mathbf{N}$ ⊨∋ q_1', as ***we*** have solved ['|-≡'] that equation herein -- ['$\mathsf{q}_{\mathbf{NN}}$ |-≡ $_{\mathbf{N}}\mathbf{Q}$'; '$_{\mathbf{N}}\mathbf{Q}$ ⊨∋ q_2']. Thereby, $\mathbf{N}$ is '''***demoted***''' [cf. Hegel] -- to the right of the '$\mathsf{q}_{\mathbf{NN}}$' ***second*** term, which denotes the as yet unsolved/unidentified/undefined ***result*** of the self-critique of $\mathbf{N}$ -- to be '$_{\mathbf{N}}$', the "mere" 'pre-subscript' of $\mathbf{Q}$, wherein $\mathbf{Q}$ solves '$\mathsf{q}_{\mathbf{NN}}$', together yielding '$_{\mathbf{N}}\mathbf{Q}$'. The latter is instantly, simultaneously ***created*** & ***promoted*** by that very ***demotion*** of the former. '$_{\mathbf{N}}\mathbf{Q}$' represents the extreme '"other"' kind of "***first***-order" 'ordinality', fruition of the self-critique of $\mathbf{N}$ ['⮐$\mathbf{N}$']. In the ***first*** term of this solution ['|-≡'], to/within the equation below, $\mathbf{N}$ is «***aufheben***»-***conserved*** ***ex***ternally. But in the ***second*** term, $\mathbf{N}$ has been «***aufheben***»-***conserved*** ***in***ternally, as indicated by ***its*** 'pre-subscript', '$_{\mathbf{N}}$', by also being «***aufheben***»-***elevated*** via $\mathbf{N}$'s ***determinate***, «***aufheben***» ***self-negation*** ['⮐$\mathbf{N}$']. These two terms, $\mathbf{N}$ ***and*** $_{\mathbf{N}}\mathbf{Q}$, are then '''added together''' ['⌐⊕¬'], i.e., '''summed''' ['⊕'], but ***non***-reductively so, that is, "***non***-amalgamatively" so [cf. Plato: «***asumbletoi***»], & also '''*oppositionally*''' so ['⌐——¬'] --

$$\mathbf{N} \longrightarrow \mathbf{NN}, \equiv \mathbf{N}(\mathbf{N}) \equiv \mathbf{N}\otimes\mathbf{N} \equiv \mathbf{N}^2 \equiv ⮐\mathbf{N} \equiv \mathbf{N} \oplus \Delta\mathbf{N} \equiv \mathbf{N} \oplus \mathsf{q}_{\mathbf{NN}} \mid\!\!-\!\!\equiv \mathbf{N} \vdash\!\oplus\!\dashv {}_{\mathbf{N}}\mathbf{Q}$$

-- i.e., modeling the ***fruition*** ['$\mathsf{q}_{\mathbf{NN}}$'], that ***we*** solve as the ***cognitive irruption*** of $_{\mathbf{N}}\mathbf{Q}$, i.e., of an ***immanent*** critique, one *of* $\mathbf{N}$, *by* $\mathbf{N}$ -- the ***self***-critique of $\mathbf{N}$ [e.g., denoted as '$\mathbf{N}(\mathbf{N})$' & as '⮐$(\mathbf{N})$', or just as '⮐$\mathbf{N}$'] -- by virtue of, and by 'self-causation' of ['⟶'], the inadvertently ***cognitively***-produced, as well as, later, ***cognitively***-detected, and, in both ***cognitions***, ***ineluctable***, '***ideo***-intra-duality' of $\mathbf{N}$'s "***first***-order", ***intuitive essence*** --

'$\mathbf{N}$'s ***ordinality***': '***quant****itative* ordinality' ***versus*** '***qual****itative* ordinality'.