F.*E*.*D*. Vignette **#21** --

On the *Nature* of the *Opposition* between the **N** *'First Standard Arithmetic'* and the **ₙQ** *'First Dialectical Arithmetic'*.

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Author's Preface. The purpose of **F**.*E*.*D*. Vignette **#21** is to clarify the nature of the relation of ***non*-dualistic**, ***dialectical opposition*** that is felt, by human 'conceptual perception', between **ₙQ**, the axiom*s*-system of the **F**.*E*.*D*. *'First Dialectical Arithmetic'*, and **N**, the *first*-order-***plus*** axiom*s*-system of the *'First Standard Arithmetic'*.

We do so by exploring the ***first triad*** of ontological *categories*, and, equally, of connoted axiom*s*-*systems* of ***dialectical arithmetic***, that arise, first, as ***opposition***, in **s**tep **1**, and, second, as ***resolution***, in **s**tep **2**, of the Seldonian ***dialectical presentation*** entitled *'The **Dialectic** of the **Seldonian Dialectical Arithmetics**'* -- as modeled by a *'**Dyadic** Seldon-Function'-based *'**dialectical meta**-equation meta-model*', a *'meta-equation'* which is itself written in the algebraic language of the **ₙQ** *'First **Dialectical** Arithmetic'* --

-- wherein '**#**' connotes the '''domain''' of the Seldonian ***dialectical* arithmetics**; of the *'**dialectical** [**meta**-]**numbers**'*.

A Note on the On-Line Availability of Definitions of **F**.*E*.*D*. ***Key Technical Terms***. Definitions of ***Encyclopedia Dialectica*** [*E*.*D*.] technical terms, including of *E*.*D*. 'neologia', are available on-line via the following URLs --

http://www.dialectics.org/dialectics/Glossary.html

https://www.point-of-departure.org/Point-Of-Departure/ClarificationsArchive/ClarificationsArchive.htm

-- by clicking on the links associated with each such term, listed, in alphabetic order, on the web-pages linked-to above.

Links to definitions of the ***Encyclopedia Dialectica*** special terms most fundamental to this vignette are as follows --

arithmetical ontological "pure" '''qualifiers'''
http://www.dialectics.org/dialectics/Glossary_files/F.E.D.,E.D._Dictionary,Definition,%27Arithmetical_Qualifier%27,28OCT2013.jpg

arithmetical "pure" '''quantifiers'''
http://www.dialectics.org/dialectics/Glossary_files/F.E.D.,E.D._Dictionary,Definition,%27Arithmetical_Quantifier%27,updated,28OCT2013,.jpg

«***aufheben***» https://www.point-of-departure.org/Point-Of-Departure/ClarificationsArchive/Aufheben/Aufheben.htm

*'**cumulum**'* http://point-of-departure.org/Point-Of-Departure/ClarificationsArchive/Cumulum/Cumulum.htm

Encyclopedia Dialectica notational conventions --

Dialectical Ideography **1** http://point-of-departure.org/Point-Of-Departure/ClarificationsArchive/EncyclopediaDialecticaNotationalConventionsIdeogramic/EncyclopediaDialecticaNotationalConventionsIdeogramic.htm

Dialectical Ideography **2** http://point-of-departure.org/Point-Of-Departure/ClarificationsArchive/EncyclopediaDialecticaNotationalConventionsIdeogramicAndPhonogramic/EncyclopediaDialecticaNotationalConventionsIdeogramicAndPhonogramic.htm

Dialectical Pictography http://point-of-departure.org/Point-Of-Departure/ClarificationsArchive/EncyclopediaDialecticaNotationalConventionsPictogramic/EncyclopediaDialecticaNotationalConventionsPictogramic.htm

Dialectical Meta-Equations **1**
http://www.dialectics.org/dialectics/Welcome_files/F.E.D.,%20A%20Dialectical%20%27%27Theory%20of%20Everything%27%27,%20Volume%200,%20FOUNDATIONS,%20Draft%20Edition%201.00%20Draft,%20last%20updated%2014JUL2012,%20E.D.%20Dialectical%20Equations%20Notational%20Conventions,%20Module%20%231%20of%202.jpg

Dialectical Meta-Equations **2**
http://www.dialectics.org/dialectics/Welcome_files/F.E.D.,%20A%20Dialectical%20%27%27Theory%20of%20Everything%27%27,%20Volume%200,%20FOUNDATIONS,%20Draft%20Edition%201.00%20Draft,%20last%20updated%2014JUL2012,%20E.D.%20Dialectical%20Equations%20Notational%20Conventions,%20Module%20%232%20of%202.jpg

''' _ev-entities_ ''' http://point-of-departure.org/Point-Of-Departure/ClarificationsArchive/Eventity/Eventity.htm

''' _evolute-ness_ ''' http://point-of-departure.org/Point-Of-Departure/ClarificationsArchive/Evolute/Evolute.htm

« _genos_ » http://point-of-departure.org/Point-Of-Departure/ClarificationsArchive/Genos/Genos.htm

''' _Historical_ or _Diachronic_ _Dialectics_ '''
http://point-of-departure.org/Point-Of-Departure/ClarificationsArchive/HistoricalDialectics/HistoricalDialectics.htm

N _first_ "_standard_" _arithmetic_ http://point-of-departure.org/Point-Of-Departure/ClarificationsArchive/NaturalNumbers/NaturalNumbers.htm

NQ '_first_ _dialectical_ _arithmetic_' http://www.dialectics.org/dialectics/Correspondence_files/Letter17-06JUN2009.pdf

'''"Seldon _Functions_''' http://point-of-departure.org/Point-Of-Departure/ClarificationsArchive/SeldonFunctions/SeldonFunctions.htm

« _species_ » http://point-of-departure.org/Point-Of-Departure/ClarificationsArchive/Species/Species.htm

'_supplementary opposition_'
http://www.dialectics.org/dialectics/Vignettes_files/Aoristos_Dyosphainthos,F.E.D._Vignette_%236,The_Dialectic_of_Oppositions,first_posted_29NOV2012.pdf

''' _Systematic_ or _Synchronic_ _Dialectics_ '''
http://point-of-departure.org/Point-Of-Departure/ClarificationsArchive/SystematicDialectics/SystematicDialectics.htm

-- and we plan to expand these public definitions resources as the _Encyclopedia Dialectica_ _Dictionary_ **Project** unfolds.

[Note: '''**Arithmetical Pure Quant**ifiers''' vs. '**Arithmetical Pure Qual**ifiers'.

In the phrase "**3** apples", we term "**3**" the "arithmetical ["pure"-]*quant*ifier", and we term "apples" the '''***ontological qual*ifier** ''' -- that is, the *kind* of thing '''*qual*ifier'''.

In the phrase "**3** pounds of apples", we term "pounds" the '*metrical*[-*unit*] *qual*ifier' -- or the '''*unit of measure qual*ifier''' -- *quant*ified by the **3**, and such that, *together*, that '[metrical] *quant*ifier **&** / "times"'Defined2 that 'metrical *qual*ifier', *jointly*, and '''"metrically"''', '*quanto*-*qual*ify', or '*qualo*-*quant*ify', the '*ontological qual*ifier', "apples".

A key use-value of the *dialectical arithmetics* is to provide shorthand, *algorithmic*, ideographical-symbolic systems for the various kinds of 'arithmetical *qual*ifiers', both with and without the co-presence of '''arithmetical *quant*ifiers''', i.e., in the form of an intermixed *dialectical progression* of both increasingly descriptively-rich '*qual*ifier-only' *dialectical arithmetics*, and also of increasingly expressive '*qualo*-*quant*ifier', or '*quanto*-*qual*ifier', *dialectical arithmetics* -- both kinds alike, and in an alternating progression, in *the dialectic of the dialectic arithmetics*.].

[Note: **Ordinal** Color-Coding. In the discourse below, we apply *relative*, visible-light-spectrum-*rising-frequency-order* '**ord**inal color coding' to assist the reader in "keeping her/his place" among the multiple, kaleidoscopically-intertwining sequences and series that constitute the 'content-structure' of that discourse. We use color-code **red**[-**ish**] for the *first* in any such sequence or series, color-code **orange** for the *second* in any such sequence or series, color-code **yellow**[-**ish**] for the *third* in any such sequence or series, color-code **green** for the *fourth* in any such sequence or series, color-code **blue** for the *fifth* in any such sequence or series, color-code **indigo** for the *sixth* in any such sequence or series, color-code **violet** for the *seventh* in any such sequence or series, and color-code '''**ultra**-**violet**''' [as in "**black light**"] for *any further* entries in any such **ord**er/sequence/succession/series/progression. The well-known 'mnemonogram' "**Roy G. Biv**" may assist the reader in tracking this '**ord**inal color coding'.].

Background. We use the symbol **N**, with "double underscore", to denote the axiom*s*-system of the so-called "**N**atural" numbers, when restricted to the axioms thereof that express its nature only in *"first-order"* logic: principally the first four Peano axioms.

That first-order system is richer, in "models", ***by far*** than the "higher-order", more axioms-rich -- more-axioms-restricted -- axioms-system of the "standard" "**N**atural" numbers, for which we use the symbol **N**, with "single underscore", given its additional axioms, including at least one axiom expressed in "second-order" logic, such as Peano's fifth axiom, the so-called '''[arithmetical] *induction*''' axiom.

The "single underscore" of the latter symbol, in contrast to the "double underscore" of the former, serves to remind us of the far 'models-richer' nature of **N** with respect to that of **N**.

That "first-order" axioms system, **N**, is so rich that it includes "*non*-standard" models of the "**N**atural" numbers.

One of those "*non*-standard models" is an ***extreme qual*itative *opposite*** of/to **N** as "standardly" interpreted -- i.e., as interpreted to be **N**.

That '''*diametrically opposite*''' system, to **N** interpreted as **N**, is ₙ**Q**, the Seldonian '*First **Dialectical Arithmetic***'.

The ₙ**Q** axiom*s*-system is not *only* a '''*diametric opposite*''' of the **N** as **N** system; it is *also* its '''*dialectical opposite*'''.

The **ₙQ** system is the *first* '''*antithesis*''' system to the **N** *first*, or «*arché*», "*thesis*" system, as generated by the Seldonian *dialectical-mathematical* '*meta-model*' *equation*, modeling the **F**.*E*.*D*., *dialectical*, *systematic* *method* *of* *presentation* of the Seldonian systems of *dialectical* arithmetic, that is, modeling the *presentation* that is entitled --

'*The* *Dialectic* *of the* Seldonian *Dialectical Arithmetics*'

-- a '*dialectical equation*' which is itself also written in the '*dialectical algebra*' of the **ₙQ** '*dialectical arithmetic*'.

The term **N** and the term **ₙQ** [thus] form the first two terms of the systems-*progression* 'poly-qualinomial' of '*The* *Dialectic* *of the* Seldonian *Dialectical Arithmetics*'. That is, they are '''*non*-amalgamatively summed''' [cf. Musès] together, and also 'antagonistically summed' together [a mental action denoted, herein, by the sign '∼⊕∼', which might well be read-off out loud as "versus" [**vs**.], or, more accurately, as "and-versus" [**&-vs**.], or as "and|versus" [**&|vs**.]], as that *speci*al case of the *gene*ric-*dialectical*, **step one**, '**thesis-system ∼⊕∼ antithesis-system**' sum, the *speci*al case that constitutes the result of **step 1** of the Seldonian *dialectical-mathematical* '*meta-model*' *equation* that encodes the *dialectical* *presentation* of that same title.

The **step s = 1** calculation and solution of that '*meta-model dialectical equation*' yields, in *shorthand* form, as the content to be presented as, **step one** of that *dialectical presentation* -- **N ∼⊕∼ ₙQ** -- an '*anti*thesis-sum', expressing an *anti*thesis so far barren of any *ex*plicit 'present-ation' of any *ex*plicit *syn*thesis.

The focus of this Vignette is *not* the full axioms-systems-progression of '*The Dialectic of the* Seldonian *Dialectical Arithmetics*', for most of the hundreds of its **s**teps that have so far been explored by the **Foundation** research collective. That is a task which has been slated, by the **F**.*E*.*D*. General Council, for a different "place" and time, a different venue.

Our main focus herein is in **step one** *only*; is the elucidation of the meaning of this *first*, *thesis*/*antithesis* *only* '*cumulum*' of that progression, **N ∼⊕∼ ₙQ**, as of the *dialectical*, '*supplementary opposition*' ['∼'] that grounds it:

$$\underline{\mathbf{N}} \sim \underline{{}_{\mathbf{N}}\mathbf{Q}}.$$

In terms of the *gene*ric '*First Dialectical Arithmetic*', the axioms-system denoted by **N** in this '*meta-model*' is a *speci*fic interpretation / application / assignment [which we "sign" by '↦'] of the *gene*ric **ₙQ** '*dialector*' $\overline{\underline{q}}_1$ --

$$\underline{\mathbf{N}} \longmapsto \overline{\underline{q}}_1.$$

The axioms-system denoted by **ₙQ** in this '*meta-model*' is a *speci*fic interpretation of / *solution for* the *gene*ric **ₙQ** '*dialector*' denoted by $\overline{\underline{q}}_{1+1} = \overline{\underline{q}}_2$ --

$$\underline{{}_{\mathbf{N}}\mathbf{Q}} \longmapsto \overline{\underline{q}}_2.$$

The **numerals** {**1**, **2**, **3**, **...** } that form the core of **N**, are *collectively* denoted, herein, by just **N**, with *no* underscore(s):

$$\mathbf{N} \equiv \{\mathbf{1}, \mathbf{2}, \mathbf{3}, \mathbf{...}\}.$$

Similarly, herein we denote the '**meta**-**numerals**' core of the $_NQ$ system by $_NQ$, in which the "single underscore" of the axiom*s*-system symbol from under the '**ɴ**' *'pre-subscript-level symbol'*, is removed, yielding just '**ɴ**' instead, and in which the "double underscore" of the '**Q**' *'script-level symbol'*, is also removed, changing it to / replacing it by a "single underscore", yielding just '**Q**' instead: $_NQ$ ≡ $\{\, \overline{\mathbb{Q}}_{\llcorner 1},\, \overline{\mathbb{Q}}_{\llcorner 2},\, \overline{\mathbb{Q}}_{\llcorner 3},\, \ldots \}$.

The Nature of the Opposition.   The ***opposition*** between the $\underline{\mathbf{N}} \longmapsto \overline{\mathbb{Q}}_{\llcorner 1}$ axioms-system of the 'first standard arithmetic', and the $_NQ \longmapsto \overline{\mathbb{Q}}_{\llcorner 2}$ axioms-system of '***non***-standard first ***dialectical*** arithmetic', is ***not*** any purportedly ***un*-*dialectical***, "**absolute**" ***opposition***; is ***not*** any supposed ***radical dualism***, forever incapable of ***dialectical synthesis***, of practical and theoretical ***transcendence***, of conceptual ***reconciliation***, of '''***complex unification***'''.

That ***opposition*** is, on the contrary, a ***reconcilable opposition***: as we said above, it is a ***dialectical opposition***.

The "pure" '''***arithmetical quant***ifiers''' of the $\underline{\mathbf{N}}$ system, and the arithmetical "pure" '''ontological ***qual***ifiers''' of the $_NQ$ system, are ***not*** actually "pure", i.e., are ***not*** devoid of any implicit '''overlap''' with, or inter-mixture of, the other.

Said another way, the "pure" '''***arithmetical quant***ifiers''' of the $\underline{\mathbf{N}}$ system, and the "pure" 'arithmetical ontological ***qual***ifiers' of the $_NQ$ system, are ***not*** "**absolutely**" distinct and dirempt, the one having "**absolutely**" no internal connexion with/to the other.

The $\underline{\mathbf{N}}$ system ***im***plicitly ***already contains*** seeds and traces of the $_NQ$ system.

We know this from our prior empirical *experience of* / '''*experiments*''' *with* that which is the *totality of reference* for this entire discourse, namely, with the *'Human Phenome'* in general, and, in particular, with its core component, our own "**N**atural" language, where various varieties of '''***qual***ifiers''', even more so than various varieties of '''***quant***ifiers''', are found to figure prominently, and, more generally, with other "**N**atural" languages, similarly populated with '''***qual***ifiers''' relative to '''***quant***ifiers''', as well as with various *"artificial"* languages, '''*engineered*''' languages, deliberately, consciously *designed* languages, such as symbolic logic, calculus, and algebra, as well as ordinary arithmetic, which have, in the past, typically featured only 'mathematical ***quant***ifiers', but some of which also feature 'mathematical ***qual***ifiers', obviously so or not.

The "artificial languages" of arithmetic, of its algebra, and of mathematics more generally, cannot fulfill their implicit mission -- that of re-encoding human "**N**atural" language(s) so as to gain all of the advantages of ideographical, algorithmic regularity, and "shorthand"-like concision -- unless they arise to full ideographical and algorithmic translation of the '''***qual***ifiers''', as well as of the '''***quant***ifiers''', of "**N**atural" languages, and of some artificial languages, alike.

The $_NQ$ system still '''***contains***''' the '$\underline{\mathbf{N}}$ interpreted as $\underline{\mathbf{N}}$' system, as indicated syntactically by the $_N$ *'pre-sub*script', subordinated to and subsumed under and before the **Q** *script-level* symbol in the $_NQ$ symbol as a whole.

[Note: We symbolize the Seldonian 'First Dialectical Arithmetic' by $_NQ$, rather than by $_NQ$, the latter with a "double underscore" for *both* the **Q** and the **ɴ**, *viz.*, the '**ɴ**', *next to* '**Q**', because the ***addition*** rules of the explicitly ***cardinal*** arithmetic of the ["***second***-order"] $\underline{\mathbf{N}}$ axiom*s*-system, not ***ex***plicit in the ["first-order"] $\underline{\mathbf{N}}$ axiom*s*-system, control the 'subscript-arithmetic' of the ***sole*** $_NQ$ operation, that of '***dialectical multiplication***', i.e., of ***the «aufheben» operation***.

‘Dialectical addition’ ‘‘‘operations’’’ in $_\mathbf{N}\mathbf{Q}$ really represent a *‘**non**-operation’* option; the ‘‘‘peaceful coexistence’’’ of distinct *‘**ord**inal, ontological **qual**ifiers’ as yet **without** the **mutual interaction** of ‘**dialectical**, **mutual**-«**aufheben**»,* ***ontological multiplication***’].

Again, the *‘**meta**-**number**’* set, or "space", of the $_\mathbf{N}\mathbf{Q}$ axiom*s*-system is denoted by --

$$_\mathbf{N}\mathbf{Q} \;\equiv\; \left\{\; \mathbb{Q}_1,\; \mathbb{Q}_2,\; \mathbb{Q}_3,\; \ldots \right\} \;\equiv\; \left\{\; \mathbb{Q}_\mathbf{n}\right\}, \text{ for all } \mathbf{n} \text{ in } \mathbf{N}, \;\equiv\; \left\{\; \mathbb{Q}_\mathbf{N}\right\}$$

-- by which we can see that each $\mathbb{Q}_\mathbf{n}$ still ‘‘‘*contains*’’’ / conserves $\mathbf{n}$, for all $\mathbf{n}$ in $\mathbf{N}$, but from which we can also see that the ‘script-level’, ‘$\mathbb{Q}$’, aspect, of each ‘meta-number’s meta-numeral’, as a whole, subordinates and subsumes the *‘**post**-**sub**script’* level, ‘ $_\mathbf{n}$ ’, aspect, of that ‘meta-number / meta-numeral’; so that the ‘$\mathbb{Q}$’ of ‘$\mathbb{Q}_\mathbf{n}$’ is thus signified as both supplementing and superseding / exceeding $\mathbf{n}$, and, thus, as superseding / exceeding $\mathbf{N}$ as well.

It is, indeed, this *dialectical*, *‘**self**-«**aufheben**» **self**-**subsumption**’* of the ***ordinal*** kind of *‘**arithmetical quant**ity’*, i.e., of the $\mathbf{N}$ of $\underline{\mathbf{N}}$ -- the kind of ***quant***ity that characterizes the "first order" $\mathbf{N}$ Peano-axiom*s* system -- that makes ***ex***plicit the hitherto only ***im***plicit ingredient of *‘**arithmetical ordinal qual**ity’* in "**N**atural" arithmetic -- the kind of ***qual***ity that characterizes the also-first-order $_\mathbf{N}\mathbf{Q}$ system.

[Note: The ‘reflexive prefix’, *‘**self**-*‘, herein, refers to the logical, conceptual action of ***the human subject*/*agent*** whose mind embodies the $\underline{\mathbf{N}}$ system, and also then *critically* reflects upon that system -- thus *critically* reflecting upon ***itself*** as embodying that $\underline{\mathbf{N}}$ system -- a mental operation which we denote by the *‘**dialectical equation-definition**’*

$$\underline{\mathbf{N}}\langle\,\underline{\mathbf{N}}\,\rangle \equiv \sim\langle\,\underline{\mathbf{N}}\,\rangle$$

-- wherein ‘$\sim\langle\,\underline{\mathbf{N}}\,\rangle$’ denotes the operation of ‘‘‘***dialectical self-negation***’’’ of $\underline{\mathbf{N}}$, i.e., denotes the *‘**self**-«**aufheben**»* ***self-negation*/*self-critique*’**, ***of*** the $\underline{\mathbf{N}}$ system, ***by*** the $\underline{\mathbf{N}}$ system itself, as that $\underline{\mathbf{N}}$ system is ‘mentally embodied’, or ‘held-in-mind’, ***by*** an $\underline{\mathbf{N}}$-cognizing human subject/agent, e.g., by the ‘presentor’, and by the ‘presentees’, or by the readers, of the first step of that ***presentation*** which is entitled *‘**The Dialectic** of the Seldonian **Dialectical Arithmetics**’*].

Indeed, $_\mathbf{N}\mathbf{Q}$ is ***born out of*** the womb of its -- ***deficient*** -- *predecessor-opposite*, $\underline{\mathbf{N}}$.

$\underline{\mathbf{N}}$ **gives birth to** its *successor*, *‘**supplementary** opposite’*, $_\mathbf{N}\mathbf{Q}$.

The $_\mathbf{N}\mathbf{Q}$ system represents what the $\underline{\mathbf{N}}$ system lacks, by virtue of $\underline{\mathbf{N}}$ containing only ***im***plicitly, ***co***vertly, what $_\mathbf{N}\mathbf{Q}$ contains ***ex***plicitly and ***o***vertly: ‘arithmetical ontological ***qual***ifier operators’.

But the $_\mathbf{N}\mathbf{Q}$ system does so at the cost of an alternative absence, of a contrary inadequacy, of a counter-balancing incompleteness, of a mirror-image imperfection, of a symmetric insufficiency -- of a *‘re-**im**plicitization’*, of a *‘re-**co**vert-ization’* -- in relation to the key deficiency of system $\underline{\mathbf{N}}$: System $_\mathbf{N}\mathbf{Q}$ lacks the ‘‘‘arithmetical ***quant***ifier operators’’’ of which system $\underline{\mathbf{N}}$ *principally* consists -- of which $\underline{\mathbf{N}}$, in an ***o***vert sense, ***exclusively*** consists.

The **N** **~** **ₙQ** '''*dialectical opposition*''' belongs to that «*species*»-category of ***the systematic dialectic of opposition-in-general*** -- of the '*dialectical speciation of the* «*genos*»-*category of opposition*' -- that we term '**supplementary opposition**', or '**progressive opposition**', which is that *dialectic's* '*dialectical synthesis species-category*', combining its '*dialectical thesis species-category*', that we term '**complementary opposition**', or '**symbiotic opposition**', with its '*dialectical antithesis species-category*', that we term '**annihilatory opposition**', or '**parasitic opposition**' [for more regarding this foundational *dialectic*, please see --

http://www.dialectics.org/dialectics/Vignettes_files/Aoristos_Dyosphainthos,F.E.D._Vignette_%236,The_Dialectic_of_Oppositions,first_posted_29NOV2012.pdf].

The *second* system in the *dialectical systems-progression* of '***The Dialectic of the*** Seldonian ***Dialectical Arithmetics***', namely **ₙQ**, thus exposes explicitly the **in**adequacy of a system -- of the ***first*** system, **N** -- of arithmetical ideographical "artificial" language, built upon **quant**ifiers, and capable of **o**vertly expressing **only quant**ifiers, such that **ₙQ** **ex**plicitly exposes that **in**adequacy of **N** by **embodying** its critique of **N** **positively**, by actually **exhibiting** a **counter-example** [cf. Imre Lakatos] to **N**, an alternative to **N**, a system which lacks **N**'s primary lack, while also fulfilling the first four, "first order" Peano postulates which were intended to establish **N**, *interpreted as* **N**, **alone**, but which, in fact, also establish **ₙQ**, as well as other "**non**-standard models" of **N**.

However, by the same token, the **ₙQ** system, as a system capable of **o**vertly expressing **only qual**ifiers, **in**capable of **ex**plicitly expressing **quant**ifiers, thereby exposes its own, complementary **in**adequacy, taking **N** initially as, apparently, the only available **counter-example** *to* its *own*, also deficient, **counter-example**; as the only available alternative to that, also deficient, alternative to **N** which **ₙQ** is.

Thus, at first, we seem to find ourselves stuck in a symmetry of insufficiencies, an impasse of contrary imperfections, a 'pendulation' back-and-forth between two alternatives whose diametrically opposite deficiencies each drive this Sisyphean circulation, away from each's *own* deficiency, toward the counter-defect of its *other*, whose '''other''' deficiency similarly drives each back again to *itself*, and so on . . ..

Of course, the way out, the consequence and result -- and the full fruition -- of this *dialectical opposition*, and of this 'pendular' alternation, between **N** and **ₙQ**, is the irruption of the conception of a *third* system in this series/progression, the irruption of their '*uni-system*', **ₙU**, which seemingly so seamlessly combines the major features of the **N** system and of the **ₙQ** system, that it makes us feel, in our 'conceptual perception', as if these features should never have been so '''*un***N**aturally''' separated in the first place, and which, in particular, features both 'arithmetical *qual*ifiers' and 'arithmetical *quant*ifiers' in a "**N**atural" *unity*, in a harmoniously integrated mutual necessity and mutual utility.

But before we turn to any focused consideration of this *synthesis*-system, born out of the inadequacies of the *opposition* of/between **N** and **ₙQ**, let us first consider, with greater scrutiny, that *opposition* itself, and consider also how that *opposition* itself is born -- how **N** gives birth to **N** **~** **ₙQ**, only thereafter moving on to consider how **N** **~** **ₙQ** gives birth to **ₙU**.

On the Nature of the *Presentational* Transition from N̲ to N̲ ~⊕~ ₙQ -- The *Dialectic* of the [Self-]Engenderment / [Self-]Construction of the ₙQ from / "by" the N̲.

At the outset of his lectures on his *dialectical* «*Logik*», Hegel spelled out, ***most succinctly***, his *gene*ric '''algorithm''' for *dialectic*(*s*), which he applied throughout his encyclopedic *opus* on his '''Philosophical Sciences''', his *dialectical*, *'''Philosophical Theory of Everything'''*, encompassing his '''Philosophical Science''' of *present Concepts* [«*Logik*»], his '''Philosophical Science''' of *present* 'exo-human' «*Natur*», and his '''Philosophical Science''' of *present* '*Spirit*-**ed**' *Beings*, of *his own time's* consciously-creative *human beings* [«*Geist*»], in the following terms [and in which I have numbered Hegel's descriptive passages, below, for greater ease-of-reference further-on] --

[1]: "The *first determination* [A.D.: ≈ '«*speci*»-fication'] is *im*mediate, while the *second* one constitutes the *sphere* posited in *its differentia*tion from the *first*."

[2]: "Within every *simple first determination*, [e.g., *ground*,] what is *determinately* different from *it*[, e.g., the *consequence* of the *ground*] *is at once also present*, but is at first *present* without yet being *ex*plicitly posited."

[3]: "In the *second determination*, *finitude* [and with it *contradiction*] again enters."

[4]: "The *third determination* is the *unity* of the *first* and the *second*, in which the *contradiction* is *resolved*. ..."

[5]: "The *progression* is as follows. The *beginning* is *simple*, *im*mediate. ... Every newly emerging concept is *more concrete*ly *determinate* than its predecessor."

[6]: "*We are always carrying everything that went before along with ourselves into what is new*, but *everything prior is*, *within what is new*, *put in its determinate place*." [A.D.: the foregoing is an assertion, by Hegel, of what we of **F**.*E*.*D* describe as the '«*aufheben*» *ev*oluteness' of *dialectic*].

[7]: "Whereas, in what preceded, each [momentarily *im*mediate] *determination* ... passed as *ultimate*, it is now *demoted* into being *only a moment* ... ."

[G.W.F. Hegel, *Lectures on Logic*, Clark Butler, translator, Introduction to the Lectures on Logic, [More Exact Concept and] Division of the [Science of] Logic, [**I**. Being], Indiana University Press [Indianapolis: **2008**], pages **79**-**80**, *italic*, underline, **bold**, shadow, and **col**or**ed** emphasis added by A.D.].

The seven 'descriptory' precepts of *dialectics* reproduced above turn out to have been instantiated in the -- opening -- portion of *'The Dialectic of the Seldonian Dialectical Arithmetics'* that we scrutinize herein, as elaborated below:

Regarding precept **[1]**: In *'The Dialectic of the Seldonian Dialectical Arithmetics'*, N̲ constitutes the "*first* determination", ₙQ the "*second* determination".

Within the present-day *'Human Phenome'*, from practical, everyday use of ordinary arithmetic, and especially for those schooled at all in mathematical theory, the "first order" system of "**N**atural" arithmetic, which we shorthand by N̲, has, precisely, '''*im*mediate''' significance and familiarity.

The ₙQ system/ideo-«*species*», on the contrary, can come to be known and familiar *to any of us* only by our first grappling with its *differentia*tion from -- its "«*differentia specifica*»" with respect to -- the N̲ system/ideo-«*species*».

Regarding precept **[2]**: The ₙQ system is already *im*plicitly present with and in the N̲ system, via the latter's internal relation to the totality of natural language(s), at the very core of the *'Human Phenome'*.

There is already a *'penumbra of knowing'*, widespread within present-day global, Terran humanity, within the *present 'Human Phenome'*, in the *present* human *'near*-consciousness', that, *gene*rically, the "**N**atural" numbers are "abstract" relative to natural languages, if not explicitly that they *abstract from* the implicit '''*qual*ifiers''' that those abstract '''*quant*ifiers''' implicitly apply-to and '''modify''', and that there is something vaguely deficient, and perhaps even '''*un*-**N**atural''', in this *'elision of the qualifiers'* as instantiated in that system of "**N**atural" arithmetic.

Seldon's accounts of his discoveries, as recounted herein, are an essay at raising this 'near-consciousness' into **ex**plicit consciousness, or, rather, are a distillation of what constituted his individual path from that *'near*-consciousness' into more **ex**plicit consciousness of this key, core aspect of *'The Modern Ideology'*; of the *modern 'Human Phenome'*; of human minds permeated by the 'exchange-value-exchange' paradigm; of the one-sided *'quant' «mentalité»*, semi-consciously mired and marooned in the *meme* of [what Marx named] '''***The Elementary Form of Commodity Value***''' -- via their incessant, intensive daily practice/reproduction of '''***the law of*** [*capital*-]*value*''', however unconsciously so.

[For a deeply insightful exploration of the formation of the *"Modern"* -- i.e., of the *'capital-value(s)-centric'* -- human «*mentalité*», as the most recent '''instar''' in the [*psycho*]*historical* development of the *'Human Phenome'*, an exploration that, unfortunately, deprives itself of even deeper -- Marxian -- insights, see: Alfred W. Crosby, ***The Measure of Reality***: *Quantification and Western Society*, **1250**-**1600**, Cambridge University Press [NY: **1997**]].

Regarding precept **[3]**: In the rise from **N** to **N ~ ₙQ**, '''*dialectical contradiction*''' enters **ex**plicitly, via '''*dialectical opposition*''', '**~**': the *'ord*inal ***quantity # ord**inal **quality*** *'intra*-duality'/*'self*-duality' '''inside''' **N** '''***ex**ternalizes*'''.

Regarding precept **[4]**: The mutual '''*dialectical opposition*''' of ₙ**Q** & **N** is resolved in ₙ**U**, which represents the conceptual ***uni***[*ti*-]***fication***, and the cartegorial ***unity***, of ₙ**Q** & **N**, as we shall see, below.

Regarding precept **[5]**: The **N** system seems ***simple***, and ***im***mediately familiar.

The concept of the ₙ**Q** system, which '''follows [from]''' the concept of the **N** system, is ***more determinate***, is ***richer*** in ***determinations***, or in '''*speci*fications''', than is that of the **N** system -- at minimum because, on its face, the ₙ**Q** system combines old ***determinations***, of the '**ₙ**' *quant*ifiers, with new ***determinations***, of the '**Q**' [*ord*inal] *qual*ifiers.

Regarding precept **[6]**: This '''***evoluteness***''' of ***dialectic*** is the reason that the ₙ**Q** axioms are designed to result in a ***dialectic series-progression*** of form, e.g. [using '**⟃**' as the "becomes" sign, or as the "goes to" sign] --

N   ⟃

N ~⊕~ ₙQ   ⟃

N ~⊕~ ₙQ ~⊕~ ₙU ~⊕~ ...   ⟃**...**

-- via the '[***Dyad***ic] Seldon Functions', rather than in a mere ['''***con*volute**'''] ***sequence progression***, of the form --

N ⟃

ₙ**Q** ⟃

ₙ**U** ⟃**....**

The *gene*ric *dialectical movement*, in both its Hegelian account, and in its Seldonian account, is thus **not** --

thesis —❸ **antithesis** —❸ **synthesis**

-- but is, on the contrary --

thesis —❸ **thesis** ~⊕~ **antithesis** —❸ **thesis** ~⊕~ **antithesis** ~⊕~ **synthesis** ....

For example, in *the cognitive context*, in reading a book, or in auditing a lecture, the '''passed''' chapters-content of that book, the '''passed'' and past passages of that lecture, do not '''**con**volutely''' vanish completely from mind once the present immediacy of our direct perception of them has passed into our past: they 'cumulate' in the mind, even if they fade in vividness with respect, especially, to new-present reading or hearing, as also with respect to also-past, but less-past passed, audited 'lectorial' utterances, or read chapters and passages.

In *the physical context*, all atoms did not vanish out of all independent existence once the first molecules formed, nor did independent molecules cease to exist once the first "prokaryotic" cells formed, and so on: *cosmological* [*meta-*]*evolution* is also '''*evolute*''', **not** '''*convolute*''' [in the main].

As a whole, *cosmological* [*meta-*]*evolution* is '*covolute*', invoking an *E.D.* term, '*covolution*', which signifies a *dialectical synthesis* of '''*evolution*''' and '''*convolution*''', encompassing the potential '''extinctions''' of *some* 'physio-ontological' categories' content, together with the '*double-*«*aufheben*»' *conservation*-into-the-present of most.

Regarding precept **[7]**: In **s**tep **0** of '*The Dialectic of the Seldonian Dialectical Arithmetics*' and because of its overall pre-programmed, 'algorithmitized' *dialectical method of exposition*, the **N** system stands alone in **ex**plicitude, and it appears, '''*moment*arily''', to constitute the *ultimate* as well as the *primary system of arithmetic*.

But the immanent, internal inadequacies of the **N** system bifurcate it, break it out into two -- i.e., it 'self-bifurcates' -- into the *first* possible *cumulum*, the two-*system cumulum*, of this *dialectical progression* of *systems* of *dialectical arithmetic*: into the **N** ~⊕~ **ₙQ** *first cumulum* of this *dialectic*.

In that process, **N** is indeed "*de*moted" from its former seeming '*ultimaticity*', as signified by the lowered, subordinate, subsumed, '*pre-sub*script' position in[to] which it is explicitly conserved and consigned in the symbol **ₙQ**, the symbol of the new '''leading''', new *apparently ultimate*, *system of arithmetic*, by means of the '**ₙ**' component of that compound ideographical symbol.

But **N** is also '''*pro*moted''', to become the '**Q**' component of that symbol as a whole, **ₙQ**.

Here we must register a critical note regarding Hegel's account of the *gene*ric *dialectical process*, as reproduced above.

The colloquial German term «*aufheben*», which, in Hegel's usage, becomes the German name for the core universal characteristic of all *dialectical process*, has three -- even concurrently-applicable -- '''moments''' to its meaning, not just the two that Hegel tends to emphasize: **(1)** to *negate*; **(2)** to *elevate*, and; **(3)** to *conserve*.

Indeed, Hegel tends to emphasize **only** moments **(1)** and **(3)**.

The '''*negation* moment''' of the «*aufheben*» *movement* is related to the '''*de*-motion from *ultimaticity* effect''' of which Hegel spoke in the lectures presented, in book form, by Clark.

The ‘‘‘*elevation* moment’’’ of the «*aufheben*» *movement* is related to the ‘‘‘***pro***-motion effect’’’ of the *__dialectic__* which we just noted above.

The ‘‘‘*conservation* moment’’’ of the «*aufheben*» *movement* is equally ingredient in both the ‘‘‘***de***-motion’’’ and the ‘‘‘***pro***-motion’’’ effects.

Hegel often addresses only the ‘‘‘*negation* moment’’’ and the ‘‘‘*conservation* moment’’’ of the «*aufheben*» *movement*, even altogether omitting to mention the ‘‘‘*elevation*’’’ aspect.

We hold this to be a mistake.

For example, we account for the formation of the $_N\mathbf{Q}$ from, and *“by”*, the $\mathbf{N}$ as follows.

The first four, “first order” Peano postulates, that form the core of the $\mathbf{N}$ axioms-system, characterize the “$\mathbf{N}$atural” numbers most explicitly as *“__ord__inal numbers”*, *__not__* as *“__card__inal numbers”*.

The ‘*__intra-dual__*’ ‘co-character’ of the $\mathbf{N}$ as, ‘‘‘simultaneously’’’, both *“__ord__inal numbers”* and *“__card__inal numbers”* is only *__im__*plicit in the $\mathbf{N}$ *of* $\mathbf{N}$.

The set or space, $\mathbf{N}$ *of* $\mathbf{N}$ is defined to be $\mathbf{N} \equiv \{ \mathbf{1}, \mathbf{2}, \mathbf{3}, \dots \}$, and thus, as a set, consists of a *__dis__*ordered plethora of *instances* of *“__ord__inal number”*, even though, in the rendering preceding, we have excerpted from that -- *__ord__*er-less -- set to present three of its members in their *__ord__*inal *__ord__*er.

But what is ‘*__ord__*inal-number-*in-__gene__ral*’*?*

What is the «*__ideo__-__genos__*» of *“__ord__inal numbers”*, *__im__*plicit in that jumble of its *instances*, $\mathbf{N}$*?*

It is the *__gene__*ral category of ‘‘‘*__ord__inal __quant__ity*’’’.

And, *__im__*plicit in the *__inherent ambiguity__* of the concept of ‘‘‘*__ord__inal __quant__ity-in-__gene__ral*’’’ is the concept of ‘*__ord__*inal-*__qual__ity-in-__gene__ral*’.

*__Im__*plicit in the *__gene__*rality of the *__quant__*itative idea of *“first”*, *“second”*, *“third”*, etc., is the *__gene__*ral *__qual__*itative idea of ‘$\mathbf{n}$th-*__ness__*’ whose *instances* are ‘the *__qual__*ity of *first-__ness__*’, ‘the *__qual__*ity of *second-__ness__*’, ‘the *__qual__*ity of *third-__ness__*’, etc.

We already know, at least latently, from our experience of natural language(s), which constitute the *totality-of-reference* for this *__dialectic of dialectical arithmetics__*, that “standard” arithmetic(s) are deficient, in terms of their ‘‘‘mission’’’, and in terms of their ‘‘‘manifest destiny’’’, in that they do not “translate” the ‘‘‘qualifiers’’’ of verbal human “Natural” language into ‘‘‘$\mathbf{N}$atural’’’-arithmetical, ‘algorithmical’ ideograms. They only “translate” the ‘‘‘quantifiers’’’.

The subtle, evanescent, attenuated form of quality that is ‘*__ord__*inal quality’, or ‘*__ord__*er quality’, is the first form of quality that can [*re*-]enter immanently into arithmetical ideography, that can [*re*-]emerge [‘*re-*’, given the *quite __ex__plicit* presence of the «**Mo**nad», or **unit**, ‘*quant*ifiable arithmetical *qual*ifier’, denoted by ‘‘‘$\mathbf{M^o}$’’’, in Diophantus of Alexandria's *circa* **250** C.E. first precursor to modern symbolic algebra, the *__Arithmetica__*, an ‘*arithmetical __qual__ifier*’ which was dropped during the Renaissance rebirth of arithmetic and algebra in pos-Dark-Ages, post-Medieval Europe] within modern arithmetic, or, i.e., within ‘algorithmic ideography’.

This, 'the arithmetical *qual*itative', can so [*re*-]emerge by means of the seeds and traces of it which are *im*plicit in the '*ord*inal *quant*ifiers', or '''*ord*er numbers''' -- in the '*consecutive*-*ord*er-numbers', to state their nature more precisely -- that *ex*plictly constitute the **N** *of* **N**, *versus* the primarily '*card*inal *quant*ifiers' that *ex*plictly constitute the **N** *of* **N**.

These '''seeds''' need only to '''sprout''', as they did in the mind of Karl Seldon, into '*ord*inal *qual*ifiers', i.e., into arithmetical, 'algorithmical', "symbolic" '*ord*inal *qual*ifier meta-number ideograms', representing the *speci*fic *qual*ities *of ord*er that inhere in the "*ord*inal numbers", or '*ord*er-numbers', of the **N** *of* **N**; in --

1, **2**, **3**, **. . .**

-- *as* 'Peanic' "*ord*inal numbers", or '*ord*er-numbers'.

In **1889**, Giuseppe Peano published his "postulates" for a deductive-logic derivation of "**N**atural Numbers" arithmetic, in his book ***The Principles of Arithmetic Presented by a New Method***. The philosopher W. L. Reese gives the original version of the four "first-order" Peano "postulates" [or "axioms"] as follows --

"P1. **1** is a ["**N**atural"] number.

P2. The successor of any ["**N**atural"] number is [also] a ["**N**atural"] number.

P3. No two ["**N**atural"] numbers have the same successor.

P4. **1** is not the successor of any ["**N**atural"] number."

[W. L. Reese, ***Dictionary of Philosophy and Religion***: ***Eastern and Western Thought***, Humanities Press [NJ: **1980**], p. **418**, square-bracketed insertions -- e.g., of '["**N**atural"]' -- and color-coded, **bold-face**, shadowed numeral **1** emphases added by A.D.].

Can you perceive, directly inspecting these **4** axiom-sentences, that, yes, they encode the **N**, but also *more than* the **N***?*

At least three theorems, all "completed" in the **1930**s, provide formal proofs implying that the "standard model" and "***non***-standard models" of these Peano-axioms' "**N**atural" Numbers arithmetic are ***inextricably*** ***coexistent*** with one another at the level of the "first order" axiomatization of arithmetics, even though these three theorems do not immanently provide a method for the ***construction*** of any of the "***non***-standard models" that they herald.

The Löwenheim-Skolem Theorem, by itself alone, implies that "***non***-standard models" of **N** inhere in the "first-order" "standard model" of "**N**atural" arithmetic.

The Gödel Completeness Theorem, which provides proof of the *semantical **completeness*** of "first-order" **N** arithmetic, given its proof of the *semantic **completeness*** of ideographical "first-order" predicate logic itself, in conjunction with the Gödel Incompleteness Theorem, which establishes the *syntactic **in**completeness of first-order* **N** arithmetic, and the *semantic and syntactic **in**completeness of all possible higher-than-first-order axiomatizations of the* **N** arithmetic, *together* imply the ***inextricable*** ***coexistence*** of "standard" and "***non***-standard" models of the Peano "**N**atural" Numbers arithmetic at the level of the "first order" axiomatization of "**N**atural" Numbers arithmetic.

Now, on their face, these four Peano postulates describe the '''**ord**er properties''' -- the '''**ord**inality''' -- of a sequence of entities that they name ["**N**atural"] "numbers".

But nothing of the **card**inal quantity features, i.e., of the '''"manyness"''' features, of the standard "**N**atural" Numbers are **ex**plicit in these four, "first order" sentence-axioms.

These four axioms are said to be "first order" because they make assertions about **only** individual "[**N**atural] numbers", and make **no** assertions about *qualities* shared by *groups* of these "logical individuals" / "numbers", i.e., by **sub**sets of the full **set N** *of* **N**, *groups* treated as '[meta-]individuals' in their own right -- assertions about which would be termed "second order".

On the very face of them, these four, "first order" Peano postulates support also *"**non**-standard models"* of the "**N**atural" numbers.

That is, they potentially describe also, besides the *"numbers"* that constitute the **N** *of* **N**, *sequences* of *other kinds* of [idea-]objects / of "logical individuals" which are **not** "**N**atural" numbers, including *sequences* of mutually **qual**itatively *different*, yet related, such objects, which are **not** "numbers" at all, in any "standard" sense of that term, **provided** that we first discard any presumptions regarding the '''**actual infinitude**''' for the counts of the entities in such *sequences*, "standard" and *"**non**-standard"* alike.

The "standard" "**N**atural" numbers do **not** form an **actually infinite sequence** in any *practical*, *empirical*, *scientifically-admissible* sense, **nor** can any **actually infinite** number of "**N**atural" numbers be actually constructed, step-by-step, by **actual** mathematicians, given the nature of physical possibility.

Practically speaking, the "**N**atural Numbers" are a **finite** sequence, albeit that their effective upper bound "**N**atural" Number, which we denote by a special symbol, $\bar{\mathbb{A}}$, is **not** always the same "**N**atural" number in every practical context.

E.g., if I am using a digital personal computer to perform arithmetic, then the **practical finite** limit of the "**N**atural Numbers" for me, in that context, is the largest "**N**atural" number which such a computer can encode, given its multi-byte "word-size", which, since **2012** C.E., has typically been about $\bar{\mathbb{A}} \equiv \mathbf{2^{64} - 1} = \mathbf{18,446,744,073,709,551,615}$.

If we can get beyond the '''"hangover"''', or '''"overhang"''', of *ancient Platonic religious mysticism* that **still** plagues modern mathematics, as we should, and as Seldon does, in his *dialectical*, *immanent critique* of the "standard" arithmetics, and, in particular, if we drop the 'contra-empirical', and thus anti-scientific, assertion of *"**actually**-existing"* *infinities*, such as the supposed Cantorian *aleph-null actual infinity* of the "**N**atural" Numbers, then we find **many** commonplace, "prosaic" examples of *"**non**-standard models"* of the Peano "first-order" "**N**atural" "numbers". [For a far-reaching critique of the prevailing, pseudo-scientific academic *religion* of *'infinitist' mysticism* in contemporary mathematics, see Brian Rotman's **Ad Infinitum**: *The Ghost in Turing's Machine* -- **Taking God Out of Mathematics and Putting the Body Back In**, Stanford University Press [Standford, CA.: **1993**]. See also Reuben Hersh, **What Is Mathematics, Really?**, Oxford University Press [NY: **1997**].].

For example, the *sequence* of the **26** letters of the English alphabet, in their standard, "alphabetical **ord**er", fully satisfy the 'finitistic' "first-order" Peano axioms [e.g., provided we assert the phrase *"except for the last* ["**N**atural"] *number"* at [the] appropriate place(s) in the Peano axioms], even though they are **not** "numbers" at all, in any "standard" sense. Just substitute the word "letter" for the word "number", and the symbol '**a**' for the ideogramic symbol "**1**", in a fully finitistic version of Peano axioms given above.

So does the *sequence* of the month-names of <u>a</u> given year -- just substitute the word "month" for "number", and the phonogramic symbol "**January**" for the ideogramic symbol "**1**", in the 'finitization' of the Peano axioms given above.

So do the day-names for <u>a</u> given week -- just substitute the phrase "day of the week" for the word "number", and the phonogramic symbol "**Monday**" for the ideogramic symbol "**1**", in a fully finitistic version of the Peano axioms.

So do the names of <u>the</u> "taxa" of the scheme of biological taxonomy, or of biological "systematics", e.g., in their systematic ***ord***er of *"as*cending"/rising inclusiveness, reversing the order of the **Domain**, **Kingdom**, **Phylum**, **Class**, **Order**, **Family**, **Genus**, and **Species** 'mnemonogram' **DKPCOFGS**, via "**D**o **K**ings **P**lay **C**hess **O**n **F**ine **G**lass **S**ets?". Just substitute the word "taxon", or "taxa", for the word "number", or the word "numbers", correspondingly, and phonogramic symbol "**Species**" for ideogramic symbol "**1**", in a finitistic version of the Peano axioms given above.

And so do the *'**ord**inal **qual**ifier **meta-numbers**'* of the $_NQ$ space of the $_NQ$ axiom*s*-system for *__dialectical arithmetic__*: another *"**non**-*standard model of the **N**atural Numbers": this time, a ***gene***ric one -- rather than a ***speci***fic one, such as is each of the earlier examples given just above.

Indeed, the first-order Peano axioms, purged of any fanciful or mystical ***infinitism***, apply to the whole domain of what Seldon calls, *<u>not</u> "**arche**onic **con**tinua"*, but *'**arche**onic con<u>secua</u>'* -- the entire realm of possible finite/realistic *sequences* of [ev]entities, such that each such *sequence* exhibits an "«***arché***»", e.g., an *'''**ultimate ancestor**'''*, a *'**start**ing [ev]entity',* but which may, at least in some cases, exhibit no pre-***ord***ained *'**stop**ping [ev]entity',* and which may thus constitute __*never*__ *"**actually infinite sequences**",* but ***ever only*** *"**potentially infinite**" sequences*, in Aristotle's sense.

*'**Ord**inal **qual**ifiers', **ord**inal **categories*** represented as/by *'**ord**inal **qual**ifier **meta**-numbers',* can thus fulfill the first four, finitistic "first order" axioms *of* $\underline{N}_{\overline{\wedge}}$ just as fully as do the *'**ord**inal **quant**ifier-**numbers**'* of the $N_{\overline{\wedge}}$ *of* $\underline{N}_{\overline{\wedge}}$ --

$$\mathbf{N}_{\overline{\wedge}} \equiv \{\mathbf{1}, \mathbf{2}, \mathbf{3}, ..., \overline{\wedge}\}.$$

The $_NQ$ *'**meta**-numbers'* of the $_NQ$ *__dialectical arithmetic__* are 'axiomatically-engineered' to ***gene***rically encompass all cases of realistic, '''finitistic''' *'**arche**onic con<u>secua</u>'* whose sequential [ev]entities are __*not*__ "standard **N**atural Numbers", but, rather, are *'''**non**-numbers''',* or "numbers" in the ***ancient***, 'qualo-quantitative' sense of «***arithmoi***»; of *'**numbers of units**'''*; but of *'''**non**-pure-**quant**itative''' **units*** -- indeed, of *'''**qual**itative **units**/[ev]entities'''* -- like the letters of <u>an</u> alphabet, the months of <u>a</u> year, the days of <u>a</u> week, <u>the</u> 'taxons' / taxa / categories of biological systematics, or the pedagogical ***sequence-****of*-***presentation*** of the *__dialectical progression__* of axiom*s*-systems for <u>the</u> Seldonian *__dialectical__* ['**#**'] ***arithmetics***, or the pedagogical ***sequence-****of*-***presentation*** of the *__dialectical progression__* of axiom*s*-systems for <u>the</u> *standard numbers'* ['**#**'] ***arithmetics***:

*'**The Goedelian Dialectic** of the Standard Arithmetics'* -- **N**, **W**, **Z**, **Q**, **R**, **C**, **H**, **O**, …

-- which we model via the *'__dialectical meta-equation__'* -- $_\#\rrbracket\!\rvert\!\mathbf{H}\!\rvert\!\underbrace{(}_{\mathbf{s}_\#} = (_\#\underline{\mathbf{N}})^{2^{\mathbf{s}_\#}}$. [for more regarding this latter example, see: **link_1**; **link_2**, **link_3**].

The $_NQ$ *'**meta**-numbers'* of the $_NQ$ *__dialectical arithmetic__* are 'axiomatically-engineered' to also ***gene***rically encompass -- to reconstruct the pasts of, and to 'pre-construct' the futures of -- cases like the historical sequence of ***the forms of human social formation***, like the historical sequence of the predominant human-social *"**relations** of production"* [cf. Marx], like the historical sequence of the predominant human-social *"**forces** of production"* [cf. Marx], like the historical sequence of emergence of the principal ***fields*** of human ideology/knowledge, which constitute a more self-conscious component of *'**The Human Phenome**',* __*and*__ like the cosmological historical sequence-of-appearance/-of "population" of the ***primary*** categories of the ***physical*** ontology of the ***cosmos***, thus constituting the Seldonian *'__Dialectical Theory of Everything__'.* [for more regarding these other examples, see: **formations**; **relations**; **forces**; **ideologies/knowledges**; **cosmos**].

Our '''self-reflection''' of **N** in the context of "natural language(s)" as *totality-of-reference* **(1)** 'conceptually perceives' the ***lack*** -- or the [psycho]historical ***loss***, in 'post-Diophantine', post-Medieval Modernity -- of 'arithmetical, algorithmic *qual*ifiers', as a ***lack*** of and within "standard"-interpretation **N**, and **(2)** 'conceptually sees' the seeds, the "wherewithal", for the redress of that ***lacking*** as residing also *immanently* within **N** itself, in the *inherent* **ambiguity** / *'intra-duality'*, of *quant*itative versus *qual*itative ***ord***inality, and, as a further, *combined result* of those two foregoing 'conceptual perceptions', **(3)** 'conceptually discovers/creates' the system of *'**ord**inal **qual**ifiers'* / of *'**qual**itative **ord**inality'*, which also turns out to be easefully open to further interpretation into systems of *'dialectical qualifiers'* -- of *'**arithmetical**, **algorithmical**, **ideographical** **dialectors**'*, i.e., into the **ₙQ** *'**meta**-**numbers**'* of the **ₙQ** axiom<u>s</u>-system [for starters].

These *gene*ric kinds of *'dialectical qualifiers'* include [the «*arché*»-]*thesis*-*qual*ifier(s), the ***full antithesis***-*qual*ifiers, the ***full synthesis qual***ifiers, the ***partial-antithesis qual***ifiers, and the ***partial-synthesis qual***ifiers, associated with the *'**Dyad**ic Seldon Function'* dialectical equation models, and/or with the *'**Triad**ic Seldon Function'* dialectical equation models.
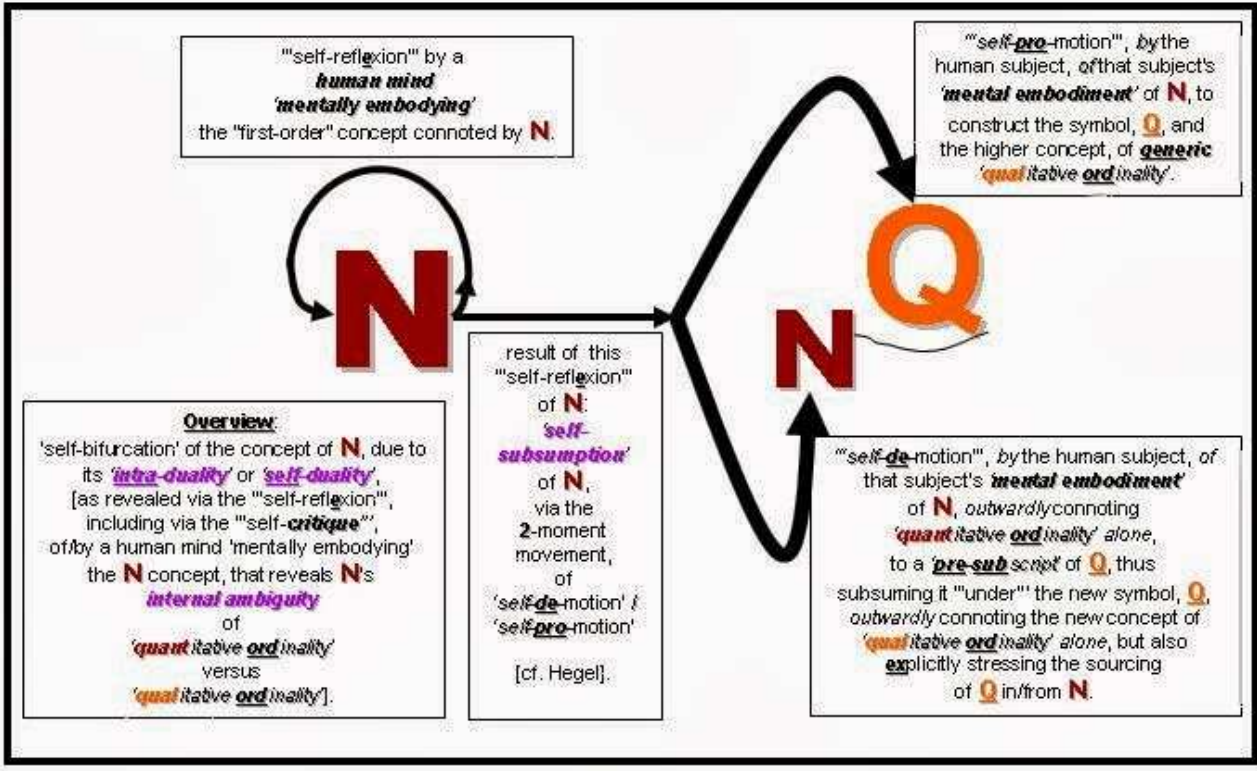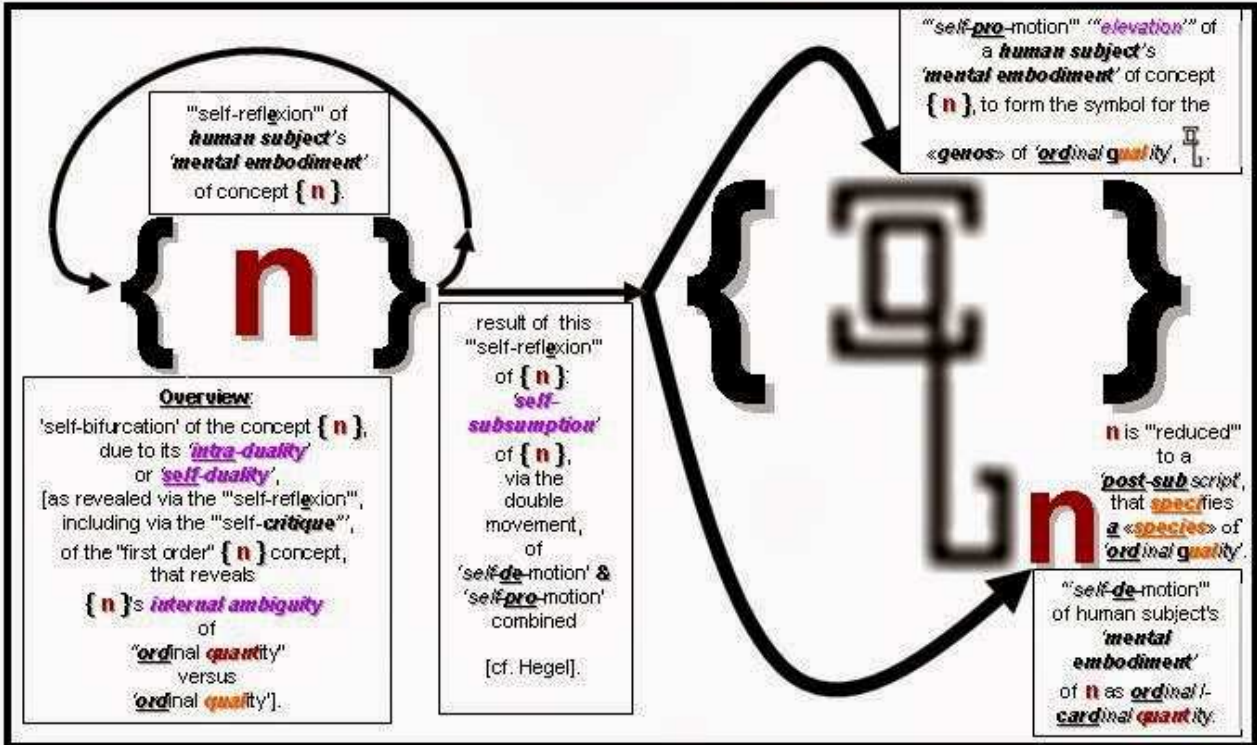
Thus, we have seen, above, both the *that*, and the *how*, of the making ***ex***plicit of this formerly ***im***plicit, '''***non***-standard''' concept of *'**ord**inal **qual**ity'*, or of *'**qual**itative **ord**inality'*, a concept that 'co-inheres' in the **N** *of* **N**, together with its ***standard*** 'co-concept' of '''***ord***inal ***quant***ity'''/'***quant***itative ***ord***inality', thereby forming, in our minds, a new space, or set, which we call --

$$\mathbf{_NQ}_{\overline{\wedge}} \;\; \equiv \;\; \{\, \mathbb{\underline{Q}}_1, \; \mathbb{\underline{Q}}_2, \; \mathbb{\underline{Q}}_3, \; \cdot\cdot\cdot, \; \mathbb{\underline{Q}}_{\overline{\wedge}} \,\}.$$

This new '''[meta-]number''' "space", or "set", is formed, in our minds, by what can be viewed, e.g., either as an *instances*-vs.-«*ideo-species*» 'self-splitting', or as an «*ideo-species*»-vs.-«*ideo-genos*» 'self-splitting', of the **{n}**, or of the **N**, respectively, which thereby 'self-bifurcate' into diverse *instances* or *speci*es of *'**ord**inal [and, ***im***plicitly or ***ex***plicitly, respectively, also ***card***inal] ***quant***ity' ***sub***scripts -- *'**post**-**sub**scripts'* or *'**pre**-**sub**scripts'*, respectively -- in both cases united with a unified, *speci*fic or *gene*ric 'ordinal **q**ual*ity* '''***script***'''-*level symbol*, which 'covers' all of these *instances*, or *speci*es, respectively, and which we denote by $\mathbb{\underline{Q}}$, or by **Q**, respectively, involving a combined, dual , split movement of 'self-***de***-motion' and 'self-***pro***-motion' of the **{n}**, or of the **N**, respectively: two related '''levels''', or '''layers''', of such '''self-bifurcation''' / 'self-subsumption', each of which can be depicted via each of the following two pictographic symbolizations, respectively:

'Dialectical «Gene»-ration' of the 'First Dialectical Arithmetic' from the 'First Standard Arithmetic'.

'Self-Construction' of the Human, Mental '''Embodiment''' of the $_N\mathbf{Q}$ from that of Peano, "first order" $\mathbf{N}$.

'''self-reflexion''' of **human subject's** 'mental embodiment' of concept **{ n }**

'''self-pro-motion''' '''elevation''' of a **human subject's** 'mental embodiment' of concept **{ n }**, to form the symbol for the «genos» of 'ordinal quality', $\mathbf{Q}$.

{ n }

{ ⬚ }

n is '''reduced''' to a 'post-sub script', that specifies a «species» of 'ordinal quality'.

Overview:
'self-bifurcation' of the concept **{ n }**, due to its 'intra-duality' or 'self-duality', [as revealed via the '''self-reflexion''', including via the '''self-critique''', of the "first order" **{ n }** concept, that reveals **{ n }**'s internal ambiguity of "ordinal quantity" versus 'ordinal quality'].

result of this '''self-reflexion''' of **{ n }**: 'self-subsumption' of **{ n }**, via the double movement, of 'self-de-motion' & 'self-pro-motion' combined [cf. Hegel].

'''self-de-motion''' of human subject's 'mental embodiment' of **n** as ordinal /-cardinal quantity.

'''self-reflexion''' by a **human mind** 'mentally embodying' the "first-order" concept connoted by **N**.

'''self-pro-motion''', by the human subject, of that subject's 'mental embodiment' of **N**, to construct the symbol, $\mathbf{Q}$, and the higher concept, of **generic** 'qualitative ordinality'.

N

$_N\mathbf{Q}$

Overview:
'self-bifurcation' of the concept of **N**, due to its 'intra-duality' or 'self-duality', [as revealed via the '''self-reflexion''', including via the '''self-critique''', of/by a human mind 'mentally embodying' the **N** concept, that reveals **N**'s internal ambiguity of 'quantitative ordinality' versus 'qualitative ordinality'].

result of this '''self-reflexion''' of **N**: 'self-subsumption' of **N**, via the 2-moment movement, of 'self-de-motion' / 'self-pro-motion' [cf. Hegel].

'''self-de-motion''', by the human subject, of that subject's 'mental embodiment' of **N**, outwardly connoting 'quantitative ordinality' alone, to a 'pre-sub script' of $\mathbf{Q}$, thus subsuming it '''under''' the new symbol, $\mathbf{Q}$, outwardly connoting the new concept of 'qualitative ordinality' alone, but also explicitly stressing the sourcing of $\mathbf{Q}$ in/from **N**.

In the first of the two illustrations above, the **1, 2, 3, . . .** that constitute the **N** ‘self-*de*mote’, in[to] «*aufheben*»-*conserved speci*ficity, to form the *plural ‘post-sub*scripts’ of the $_N\mathbf{Q}$ -- of the $\mathbf{q}_1$, $\mathbf{q}_2$, $\mathbf{q}_3$, **. . .**.

But the **1, 2, 3, …** also ‘self-*pro*mote’, in[to] «*aufheben*»-*conserved*, «*aufheben*»-*elevated* greater] *gene*rality, in and by forming the *singular **script**-level* $\mathbf{q}$, and **Q**, which denote the «*ideo-genos*» of ‘*ord*inal *qual*ity’ -- ‘*ord*inal-*qual*ity-in-*gene*ral’.

As a result, each member of the new space, or set, $_N\mathbf{Q}$, of ‘*specific ordinal quality*-denoting symbols’ -- of symbols which are ‘*meta-numerals’* with respect to the **N** “*numerals”* -- and which can be expressed *gene*rically by the ‘‘‘algebraic’’’ symbol $\mathbf{q}_n$, where **n** is a[ny] member of **N**, is a combination of two symbolic elements or determinants:

(1) at its ‘*script* level’, it contains the *gene*ric symbolic element ‘$\mathbf{q}$’, which stands for ‘*ord*inal-**q***ual*ity-in-*gene*ral’, and

(2) at its ‘[*post*-]*sub*script level’, it contains a ‘*speci*fic’ value of the ‘*speci*fic symbolic element ‘**n**’, e.g., any value from the space **1, 2, 3, . . .**, as a ‘*speci*fier’, one which ‘*speci*fies’ the ‘*speci*fic *ord*inal *qual*ity’ for which $\mathbf{q}_n$ stands.

E.g., $\mathbf{q}_1$ stands for **any** *first category*, or *first system*, etc., in any *sequence*, *succession*, or *progression* of *categories*, or of *systems*, etc.

E.g., $\mathbf{q}_2$ stands for **any** *second category*, or *second system*, etc., in any *sequence*, *succession*, or *progression* of *categories*, or of *systems*, etc. E.g., $\mathbf{q}_3$ stands for **any** *third category*, or *third system*, etc., in any *sequence*, *succession*, or *progression* of *categories*, or of *systems*, etc., etc.

We can compare/contrast/relate this $_N\mathbf{Q}$ space of ‘*qual*itative *ord*inalities’ to the **N** *of* **N** space of ‘*quant*itative *ord*inalities’ from which it, per above, arises, as follows, using ‘$\longmapsto$’ as ‘generic interpretation/assignment’ symbol --

‘*Quant*itative *Ord*inality’: *First* $\longmapsto$ **1**st; *Second* $\longmapsto$ **2**nd; *Third* $\longmapsto$ **3**rd, etc. **...**;

‘*Qual*itative *Ord*inality’: **1***st-ness* $\longmapsto$ $\mathbf{q}_1$; **2***nd-ness* $\longmapsto$ $\mathbf{q}_2$; **3***rd-ness* $\longmapsto$ $\mathbf{q}_3$, etc. **...**;

This view implicitly holds that the ‘*ordinal qual*ifier’ categories of *gene*ric ‘*first-ness*’, ‘*second-ness*’, and ‘*third-ness*’, etc., have a *gene*ric, universal, empirical, scientific meaning, to a degree that the potential ‘*cardinal qual*ifier’ categories of *gene*ric ‘many-*ness*’, *gene*ric ‘*one-ness*’, ‘*two-ness*’, and ‘*three-ness*’, etc., do **not** have in any way. The further -- now **ex**plicitly *dialectical* -- interpretation of these *gene*ric ‘*ordinal qual*ifier’ *category-symbols*, e.g. --

1*st-thesis* $\longmapsto$ $\mathbf{q}_1$;

1*st-antithesis* $\longmapsto$ $\mathbf{q}_2$;

1*st-synthesis* $\longmapsto$ $\mathbf{q}_3$

-- as per the *Dyad*ic and *Triad*ic ‘‘‘Seldon Functions’’’ alike, constitutes a *second* layer of their Seldonian interpretation: the **ex**plicitly *dialectical* layer of that interpretation.

Before their construction as explicitly *'dialectical qualifiers'*, the $\boxed{\top}_\mathbf{n}$ are first constructed as *'ordinal qualifiers'*. Thus, we are holding that --

N *of* **N** $\equiv$ { *first*, *second*, *third*, **. . .** } [standard interpretation]

-- whereas --

$_\mathbf{N}\mathbf{Q}$ *of* $_\mathbf{N}\mathbf{Q}$ $\equiv$ { *'first-ness'*, *'second-ness'*, *'third-ness'*, **. . .** }

-- so that --

first $\longmapsto$ **1st** $\equiv$ *'ord*inal *quant*ifier' number **1**, *element* of **N** *of* **N**;

second $\longmapsto$ **2nd** $\equiv$ *'ord*inal *quant*ifier' number **2**, *element* of **N** *of* **N**;

third $\longmapsto$ **3rd** $\equiv$ *'ord*inal *quant*ifier' number **3**, *element* of **N** *of* **N** ...

-- whereas --

1*st-ness* $\longmapsto$ $\boxed{\top}_\mathbf{1}$, *'ord*inal *qual*ifier *meta*-number', *element* of $_\mathbf{N}\mathbf{Q}$ *of* $_\mathbf{N}\mathbf{Q}$;

2*nd-ness* $\longmapsto$ $\boxed{\top}_\mathbf{2}$, *'ord*inal *qual*ifier *meta*-number', *element* of $_\mathbf{N}\mathbf{Q}$ *of* $_\mathbf{N}\mathbf{Q}$;

3*rd-ness* $\longmapsto$ $\boxed{\top}_\mathbf{3}$, *'ord*inal *qual*ifier *meta*-number', *element* of $_\mathbf{N}\mathbf{Q}$ *of* $_\mathbf{N}\mathbf{Q}$ ....

In, for example, '$\boxed{\top}_\mathbf{2}$' we have thus arrived at a *gene*ric symbol, a symbol for an 'ideo-«*genos*»', namely, the symbol '$\boxed{\top}$', standing by/standing next to, but also standing apart from -- in this notational convention, standing **above** -- a *speci*fic symbol, a symbol for an 'ideo-«*species*»', namely, the symbol '$_\mathbf{2}$'.

An 'ideo-«*genos*» signifier' standing over and above an 'ideo-«*species*» signifier' -- an 'ideo-«*genos*» category-unit' '''over''' an 'ideo-«*species*» category-unit'; a «*genos*» next to / '''over''' a «*species*».

Ring any bells*?*

For us, this rings out resoundingly about Plato's «*arithmoi eide-tikoi*», a Greek term which translates to the English *'assemblages of eide-units'*, i.e., of '*ιδεα-units'*, of *class*[ification]-units, of *category*-units, or, which translates, "equivalently", to the English '''numbers of ideas''' -- the very heart of Plato's, originating Occidental version of «*dialektikê*».

Remember how Plato describes «*dialektikê*», later on in the progression of his dialogues --

"   STRANGER: Well, now that we have agreed that *the kinds* [*the* «*gene*» -- A.D.] stand toward one another in the same way as regards blending, is not *some science* needed as a guide on *the voyage of discourse*, if one is to succeed in pointing out which *kinds* are consonant, and which are incompatible with one another – also, whether there are certain *kinds* that pervade them all and connect them so that they can blend, and again, where there are *divisions* [*separations*], whether there are certain others that traverse *wholes* and are responsible for the *division*?

       THEAETETUS: Surely *some science* is needed – *perhaps the most important of all*.

       STRANGER: And *what name shall we give to this science*? Or – good gracious, Theaetetus, have we stumbled unawares upon *the free man's knowledge* and, in seeking for the Sophist, chanced to find *the philosopher* first?

       THEAETETUS: How do you mean?

       STRANGER: *Dividing according to kinds*, not taking the same *form* [«*eidos*»; «*ιδεα*» -- A.D.] for a different one or a different one for the same – is not that the business of *the science of dialectics*?

        THEAETETUS: Yes.

        STRANGER: And the man who can do that discerns clearly *one form* everywhere extended throughout many, where each one lies apart, and *many forms*, different from one another, embraced from without by one *form*, and again *one form connected in a unity through many wholes*, and *many forms*, entirely marked off apart. That means knowing how to *distinguish*, *kind by kind*, in what ways the several *kinds* can or cannot combine.

        THEAETETUS: Most certainly.

        STRANGER: And the only person, I imagine, to whom you would allow this *mastery of dialectic* is the pure and rightful *lover* [the «*philo*» -- A.D.] *of wisdom* [of the «*sophia*»: of *skill*; *deeper knowledge* -- A.D.]."

[E. Hamilton, H. Cairns, editors, *Plato*: *The Collected Dialogues*, Princeton University Press [Princeton, New Jersey: **1989**], pp. **998-999**, *Sophist*, **253b** – **254d**, *emphases added by* A.D.].

By the "kinds" referenced in the above extract, let us understand the «*eide-gene*» categories, or «*eide-hyper-species*» categories, and, also, their implied "*sub*-kinds", or «*eide-hypo-gene*», their subsumed, «*eide-species*» *sub*-categories.

Each $_NQ$ 'meta-numeral', *gene*rically denoted by '⊡$_n$', thus represents, in itself, a minimal -- a one-«*eide-genos*»-over-one-«*eide-species*» -- «*arithmos eide-tikos*», or '*assemblage* of categories-as-units', and of their *sub*-categories-as-their-*sub*-units.

That is, each $_NQ$ '*meta-numeral*', of the *gene*ral form '⊡$_n$', for an **n** in **N**, is an *assemblage* of the minimum, two, '*ιδεα*-units' required to constitute an «*arithmos*».

One of the two, '⊡', represents the «*genos*» of the '*ord*inal *qual*ity' categories, a «*genos*» which has many «*species*».

This «*genos*» symbolic element, '⊡', thus '«*gene*»-rizes' the '⊡$_n$' symbol as a whole.

The other of the two, '$_n$', represents just one particular «*species*» of the «*genos*», the "**n**th" «*species*», and thus, this other symbolic element, '$_n$', *when it takes on a specific value* from the set **N**, '''*speci*fies''' the '⊔$_n$' symbol as a whole.

Thus, in the case of the 'meta-numeral' '⊔$_2$', the whole symbol denotes *gene*rically the 'meta-number', the '*ord*inal *qual*ifier', for the <u>**second**</u> [ev]entity in *any* succession of [ev]entities, i.e., for the *gene*ral *qual*ity of <u>*ord*inal</u> '*second*-<u>*ness*</u>'.

Thus, each **_NQ** '*meta*-**n**umeral' -- of the *gene*ral form '⊔$_n$', for some *speci*fic **n** in **N** -- "intends" an indefinitely-large *assemblage* [«*arithmos*»] of interpreted ***instances*** of the '**n**th [ev]entity of *a speci*fic succession', so that each such '*meta*-**number**' **unit** of **_NQ**, namely, '⊔$_n$', given that each one of those '⊔$_n$' **units** is *made up out of* a <u>*homo*</u>geneous *multiplicity* of the [summed] **1 unit**(s) of **N** -- for example, ⊔$_2$ = ⊔$_{1+1}$, ⊔$_3$ = ⊔$_{1+1+1}$, and ⊔$_4$ = ⊔$_{1+1+1+1}$, etc. -- so that each '*meta*-**number**' **unit** of **_NQ**, is a '*meta*-**unit**', or '*meta*-«*monad*»', of the ["**n**umber"] **unit**(s), or «*monad*»(s), of **N**, <u>***except for***</u> that singular **unit** of **_NQ** which represents the *gene*ric «*arché*», or ***ultimate <u>starting-point</u> category***, ⊔$_1$, which "contains" <u>***not***</u> a "*mulitplicity*", but only a *single*, *solitary* instance, of the **unit** of the **N**, namely, of **1**.

The Platonian «*arithmoi <u>eidetikoi</u>*», precursors to the categories or "***Concepts***" [«***Begriff***»] of Hegel's «***Logik***», and of Hegel's *System* of the '''Philosophical Sciences''' *entire*, including the categories of Hegel's «***Natur***» and "***Spirit***", or «***Geist***», constitute the original stage and paradigm of «*<u>dialektikê</u>*» in the entire Occidental tradition and history of <u>*dialectic*</u>: the historical «*<u>arché</u>*» form and formulation of all Occidental «*<u>dialektikê</u>*», despite, and notwithstanding, the prelude to this «*<u>dialektikê</u>*» in Zeno's method of "indirect", or «***reduction ad absurdum***», proof [*proof by* [deduction to a propositional] *contradiction*], to which Aristotle alludes, and in the 'Socratean' «*elenchus*» strategy of dialogue.

The Seldonian '***first <u>dialectical arithmetic</u>***' is thus precisely a <u>***dialectical arithmetic***</u> in the following deep and historically telling sense: each and every '*meta*-**numeral**' of the **_NQ** space of the **_NQ** axiom<u>*s*</u>-***system of arithmetic*** for '<u>***dialectical meta*-numbers**</u>' is, by itself, already a ***minimized*** «***arithmos <u>eide</u>tikos***» <u>*dialectic*</u> [albeit a '<u>***trans***</u>-Platonian' one], a «*genos*»|«*species*», 'ideo-systematic', 'ideo-classificatory', 'ideo-taxonomic' <u>*dialectic-in-itself*</u> --

⊔$_n$, **n** in **N**

-- or, using Seldon's '*qual*itative fraction', '<u>***non***</u>-amalgamative, *qual*itative division' notational convention, '⊔/**n**' --

«<u>*genos*</u>» level:

  *qual*itative division [«*diairesis*»] **bar** / '''**separatrix**''':   _____

«<u>*species*</u>» level: **n**

Still, one further layer of interpretation of the **_NQ** is needed to constitute them as the ideograms for a fully <u>***dialectical ideography***</u>, as the '*meta*-**numbers**' needed for an '''***arithmetic of <u>dialectic</u>***''' -- a procedure to which we already alluded, in passing, above.

That further layer of interpretation is the layer supplied in conjunction with the '''Seldon Functions'''.

Both the *'Dyadic Seldon Function'* and the *'Triadic Seldon Function'* can apply/interpret/assign ⌐$_1$, the *'ord*inal *qual*ifier' for *gene*ric *'first-ness'*, to serve as the *qual*ifier ideogram for the *gene*ric *'first thesis'* ontological category, i.e., for the «*arché*»-*thesis* ontological category, of *dialectical categorial progressions* in *gene*ral.

Both can also apply ⌐$_2$, the *'ord*inal *qual*ifier' for *gene*ric *'second-ness'*, as the *qual*ifier ideogram for the *gene*ric *'first anti-thesis'* ontological category, or *'first contra-thesis'* ontological category, for *any dialectical categorial progression*.

Likewise, both can apply ⌐$_3$, the *'ord*inal *qual*ifier' for *gene*ric *'third-ness'*, to serve as the *qual*ifier ideogram standing for the *gene*ric *'first syn-thesis'* ontological category, or *'first uni-thesis'* ontological category, for *dialectical categorial progressions* in *gene*ral.

And so on, for those 'higher *ord*er-*qual*ity', 'higher-*ord*inal' terms / '*qual*ifiers' -- those for which the further Seldonian *dialectical* interpretations, per the '*Dyad*ic Seldon Function' *versus* per the '*Triad*ic Seldon Function', *diverge*.

Of course, the foregoing *ac*count of the conceptual *gene*sis, or conscious construction, of the $_N$**Q** system in the process of formation of the *dialectical opposition* between **N** and $_N$**Q** -- **N** ∼ $_N$**Q** -- does not attempt to *re*count the actual "fits and starts", the '''trials and errors / tribulations''', and the «*sturm und drang*» that attended the actual, historical discovery of the $_N$**Q**, *especially* given the evidence that Plato's, "«*asumbletoi*»" ['*un*addable'], «*arithmoi eidetikoi*» *dialectic* already constituted a partly still '''Parmenidean''', and 'proto-algorithmical', 'pre-ideographical precursor' to Seldon's -- also «*asumbletoi*» -- **1996** discovery, occurring some ∼ **2,400** years later.

Rather, this account *presents*, **not** the actual '''method *cum accident'''* of their historical discovery, but a pedagogically designed conceptual reconstruction and reproduction of that discovery, pruned, cleansed, 'parsimonized', and tidied of dead ends and set backs, etc., and augmented with later-arisen insights, via the advantages of 'retro-perspective'.

The actual, vicissitudinous, somewhat convoluted course of Seldon's discovery of the $_N$**Q** is recorded in the **Foundation** archives, in Seldon's research notes from the *circa* **1996** C.E. temporal locus of his first "full" discovery of them.

The General Council of the **Foundation** is developing plans to make these research archives accessible to the public, at the *psychohistorically*-appointed time.

Conclusion. At its deepest root -- the deepest root that we have so far been able to discern -- the conceptual origin of both **N** and $_N$**Q** is *ordinality*, is, e.g., the *consecutive ord*er of *abstract time*, or of "pure" *time* [cf. Hamilton], of *'gene*ric *temporality'*, or of *'gene*ric *historicality'*, and the deepest root of the subtle *dialectical opposition* between **N** and $_N$**Q** -- **N** ∼ $_N$**Q** -- is the *internal opposition within* **N** itself, is the 'in*tra*-duality' or '*self*-duality' "inside" **N** -- **N** # **N** -- between *'ord*inal *quant*ities'/'*ord*er *quant*ities' -- '*quant*itative *ord*inality' -- *and* '*ord*inal *qual*ities'/'*ord*er *qual*ities'; '*qual*itative *ord*inality', a '*self*-duality' already *im*plicitly present *within* the 'human-phenomic' concept of **N** itself.

Given the notational convention that '**a** ⊐ **b**' signifies '**a** *im*plicitly contains **b**', we thus have --

N ⊐ **N** ∼ $_N$**Q**   ⬌   **N** ⊐ *'quant*itative *ord*inality' ∼ *qual*itative *ord*inality'.

Thus, the **N** ∼ **ₙQ** *dialectical opposition* is an '**ex**plicitization' and '**ex**ternalization' of a conceptual '**intra**-duality', a conceptual '**self**-duality', or a '''**dialectical** conceptual **self-** / **internal contradiction**''' [denoted by '**#**'], i.e., is an **immanent opposition**, a '''**self-opposition**''', a '**self-antithesis**', that is **inherent** in, and/or that has been '''**psychohistorically**''' **in**ternalized within, and that is thus now **in**ternal to, and **im**plicit within, the "lawful", predictable '''**psychohistorically**''', '**human-phenomic**' **construct** connoted by **N**, *even "all alone" / "by itself" / "in itself"*.

Given the notational convention that '**a** ⊑ **b**' signifies '**b** **im**plicitly contains **a**', we have --

'**quant**itative **ord**inality **#** **qual**itative **ord**inality'   ⊑   **N**, and --

'**quant**itative **ord**inality **#** **qual**itative **ord**inality'   ⟷   **N** **#** **N**   ⊑   **N**.

Fruition of the N ∼ **ₙQ** *Opposition* in a **Dialectical Synthesis System**, ₙ◊̂_QN ≡ **ₙU**, embodying their *Complex Unity*. [Fuller Title: **Fruition of the N** ∼ **ₙQ** *Dialectical Opposition* in a **Dialectical Synthesis** axiom**s**-*System*, ₙ◊̂_QN ≡ ₙ◊̂_U ≡ **ₙU**, embodying a *Complex Uni[t-i]fication* of the **ₙQ** axiom**s**-*System* and the **N** axiom**s**-*System*].

As a **gene**ric '**category-level unit**', or '**category** **as** a **unit**/«**monad**»', each ⬚̣ₙ, especially once "interpreted" or "assigned" for a **particular** *dialectical categorial progression*, stands, **im**plicitly, for a '[**meta**-]**unit**', or for a '[**super**-]**unit**', which is, in turn, **made up out of** a typically **heterogeneous** **multiplicity** of [**sub**-]**units**, i.e., the '''**elements**''', '''**logical individuals**''', '''**instances**''', or '''**speci**mens**''', that '''instantiate''', and '''substantiate''', e.g., the '''**speci**es''', ⬚̣ₙ.

These [**sub**-]**units** are **not** "**identical**" to one another, for how could they be "**identical**" and still be distinct and distinguishable / **heterogeneous?**

On the contrary, these [**sub**-]**units** are, instead, mutually-**similar** to one another, e.g., in an '**un**scaled **fractal**' sense, while also exhibiting "variation" among themselves -- "individual differences".

These **(1)** [**sub**-]**units** are also the **base-level**, or **most-concrete**, **foundational qual**itative **instances** -- the **qual**itative "**logical individuals**", the **qual**itative "**members**", the **qual**itative **cases**, the **qual**itative **specimens**, the **qual**itative **examples** -- of some, '**relative**ly' more **gene**ral, "**kind**" [cf. Plato] -- e.g., typically, **(2)** of a «**species**», but also possibly **(3)** of a [**super**⁰-]«**genos**», or, equivalently, of a '**super**¹-«**species**»', or **(4)** of a '**super**²-«**species**»', or, equivalently, of a '**super**¹-«**genos**»', or **(5)** of a '**super**²-«**genos**»', or **(6)** of a '**super**³-«**genos**»', or **(7)** of a '**super**⁴-«**genos**»', etc. -- a plurality of **base-level** [**sub**-]**units** for which that **category** / «**arithmos**» ["**number** of **units**"] / '''**ideographical population NAME**''', ⬚̣ₙ, stands, **collectively**, in a '[**meta**-]**unit**ary', **uni**vocal way.

That is, an ontological *category connoted by / associated with / assigned to / interpreted as* a given $_N\underline{Q}$ «*aufheben*»-operator / '*meta*-**N**atural *meta*-*num*ber' / '*meta*-**N**atural *meta*-*numeral*' symbol, of the *gene*ral form $\overset{Q}{\underset{n}{\boxminus}}$, is, in **F**.*E*.*D*. usage, *im*plicitly understood to stand for an «*arithmos*» in something like the ancient Greek sense, i.e., an «*arithmos*» *OF qualitative* «*monads*», or *OF qualitative units*, a '''*number*''' *OF* such *ultimate units* [*"ultimate" only relative to* the *universe-of-discourse* in play, *not* in *any absolute / reductionist* sense].

We argue, in this section, that the mentally-perceived *opposition* -- in the mind(s) of the 'presentee(s)' / reader(s) / 'thinker(s)-through' of this *presentation*/*meta-model* -- of the $_N\underline{Q}$ axiom*s*-system, *vs.*/*to* the **N** axiom*s*-system, once they both come into 'co-*present* co-existence' within such (a) perceiving mind(s) -- is typically *followed*, *intuitively*, in such (a) mind(s), by their also mentally-constructed *mutual interaction*/'*ideo-ontological multiplication*', denoted $_N\underline{Q} \otimes \underline{N}$.
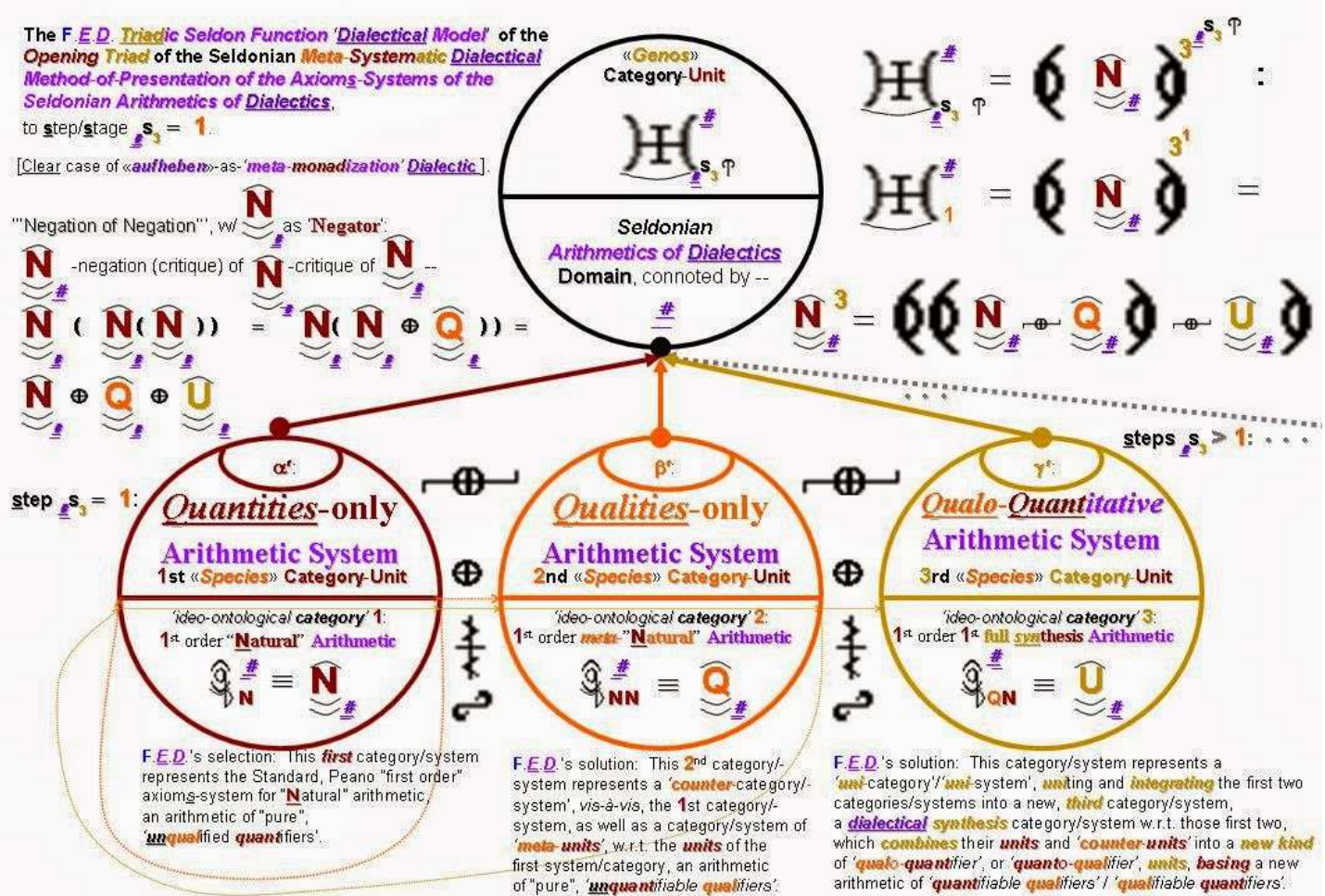
The consequent *operation* / '''[*re*-]*flexion*''', *of* the $_N\underline{Q}$ system-*as*-«*aufheben*»-*operator*, *upon* the **N** system-*as*-«*aufheben*»-*operand*, or the unique mutual '''multiplication''', mutual '''function-ing''', or *mutual* «*aufheben*»-*operation*/*mutual* '''[*de*-]*flexion*''', *speci*fic to the $_N\underline{Q}$ system/operator together with the **N** system/operator, in their *mutual interaction* with/upon one another, $_N\underline{Q} \otimes \underline{N} \equiv {}_N\underline{Q}(\underline{N})$, *includes*, *at* the [*relative*] *base-level* for their two, respective, *universes-of-discourse*, the *base-level mutual opposition* of the '*meta*-*numerals*' $\overset{Q}{\underset{n}{\boxminus}}$ *vs.* the *numerals* **n**.

This *mutual interaction*, we hold, logically, *intuitively*, gives rise, in the perceiving human mind, to a new *"kind"* , i.e., to a new, *third*, idea/system; rising up to a new, expanded '*ideo-ontology*', of superseding, succeeding, supplementary, '''*higher*''' '*meta*-*meta*-*numerals*' -- *higher* in the sense of being *richer* in expressive power, in capability for *ex*plicitly *expressing more kinds of determinations* than can either of the two previously-evoked systems/languages -- and thus to '*meta*-*meta*-*numerals*' which escape / transcend that [thus *now former*] *base-level vs. meta-level mutual opposition*.

We are describing, here, a human-mental process, whereby the mind of the 'presentor', **&**, if the *presentation*/'presentor' is successful, also the mind(s) of the 'presentee(s)' -- all constituting the human subjects/agents who are willingly/'will-fully' conducting these mental processes -- 'mentally-embody'/'mentally simulate', this *antithesis*, this *opposition* of the outward meaning of $_N\underline{Q}$, and/with/against the outward meaning of **N**, and are thereby provoked to *combine* these two, mutually-inadequate, *dialectical supplementary opposites*, by means of their '*dialectical multiplication*'/*interaction*, or mutual «*aufheben*»-*negation*, connoted by the 'definitionary equation' $_N\underline{Q} \otimes \underline{N} \equiv {}_N\underline{Q}(\underline{N}) \equiv {}_N\underline{Q} \underline{of} \underline{N}$.

This human, mental process can be illustrated, as follows, in the '*dia*[*lecto*]*gram*' below.

1st _Triad_, Seldonian _Meta-Systematic_ _Dialectical_ _Presentation-Model_ for the Seldonian _Dialectical Arithmetics_ -- '«_arché_»-_category_'; _first_ [full] '_contra-category_'; _first_ [full] '_uni-category_'

The F._E.D._ _Triadic_ _Seldon Function_ '_Dialectical Model_' of the _Opening Triad_ of the Seldonian _Meta-Systematic_ _Dialectical Method-of-Presentation_ of the _Axioms-Systems_ of the Seldonian Arithmetics of _Dialectics_, to step/stage $\underline{s}_3 = 1$.

[Clear case of «_aufheben_»-as-'_meta-monadization_' _Dialectic_].

'''Negation of Negation''', w/ as '_Negator_':

Genos Category-Unit

Seldonian **Arithmetics of Dialectics** Domain, connoted by --

---

These new, higher, '_meta-meta-numerals_' are of the _gene_ric form $u_n \otimes \overset{o}{\underline{u}}_n$, or, more simply, of _gene_ric form $u_n \overset{o}{\underline{u}}_n$.

In the latter form, the **ex**plicit '$\otimes$' sign for the '_gene_ralized / [sometimes] **non**-amalgamative _multiplication operation_', herein applied in this new, **non**-classical _system of arithmetic_, is simply understood to be indicated, **im**plicitly, by _mere_ **juxtaposition**, of u_n and/with $\overset{o}{\underline{u}}_n$, _alone_ -- as, in _classical_ algebra, for the _classical_ multiplication operation -- wherein mere 'juxtapositioning' of algebraic **variables**, e.g., with other(s) such, or with numerals / numeric **constants**, without any intervening/mediating sign -- e.g., without any '×', or "times", sign -- signifies their multiplication, _all by itself_.

Together, u_n and $\overset{o}{\underline{u}}_n$ constitute the '''_complex_''' **u**nits, or the '''_compound_''' **u**nits -- _compound_ed of _both_ a **quant**ifier, u_n, _and_ a **qual**ifier, $\overset{o}{\underline{u}}_n$; of a u_n **quant**ifier '''_ideo-**gram-matically** modifying_''' ['''**quant**ifying'''] a $\overset{o}{\underline{u}}_n$ **qual**ifier -- that constitute the new, higher, '_meta-meta-number_' "space", which we of **F.**_E.D_ denote by $_N\underline{U}$, and of the new, higher axiom**s**-system which we denote by $_N\overset{\mathclap{\Diamond}}{\Diamond}_{\underline{QN}}$, or by $_N\overset{\mathclap{\Diamond}}{\Diamond}_{\underline{U}}$, or by $_N\underline{U}$.

The '^{o}' '"degree-sign"' superscript of the u^o_n *qual*itative-**u**nit-*qual*ifier -- which harks back all the way to the sign for the **ex**plicit *quant*ifiable '«M^o *nads*» *qual*ifier', written [approximately] as '"M^o"', used in Dyophantus's *circa* **250** C.E. founding treatise on "symbolical" algebra, the «***Arithmetica***» -- signifies that this new **u**nit-*qual*ifier, 'u^o_n', is an '*add*able [Plato: «***sumbeltoi***»], ***quant**ifiable* *qual*ifier', **un**like the old '⊔$_n$', '**un**quantifiable, **un**addable [Plato: «***a**sumbeltoi*»] *qual*ifier' **units** of the $_N\mathbf{Q}$ *dialectical* **arithmetic**.

In these new, symbolic/ideographical/syntactical '***complexes***' / '***compounds***', the u^o_n component, or '"*factor*"', denotes the 'arithmetical *qual*ifier' for the [relative] **base-level** *qual*itative **units** / «***monads***» **of** a given **kind**, **of** a given «***arithmos***», **of** a given ***ontological category***, **NOT** for the *category* itself -- **no longer** for their *category* itself **as** a **unit**, for which a corresponding '⊔$_n$' would stand. If, in the $_N\mathbf{Q}$ -language, ⊔$_1$ is *assigned to*/*interpreted as* standing for the **kind** [e.g., «***genos***»]/***ontological category***/«***arithmos***» of **quarks**, which have **6** *sub*-**kinds** [e.g., «***species***»], then the $_N\mathbf{Q}$ language's ⊔$_1$ corresponds to **6**$\underline{u}^o_1$ in the $_N\underline{\mathbf{U}}$-language.

The u_n component / '"*factor*"' denotes the '**u**nit-*ic* /«***monad***»-*ic* arithmetical *quant*ifier', one that '"***modifies***"'/-*quant*ifies that *qual*ifier as a *qual*ifier denoting a **gene**ric [*relative*] **base-level individual** [*sub*-]**u**nit / [*sub*-]«***monad***» -- *relative* to the ***category***, ⊔$_n$', as their [*meta*-]**unit** / [*meta*-]«***monad***» -- by *speci*fying the *quant*itative determination, the ***card**inal* *quant*ity determination, u_n, of the **u**nits-*qual*ifier *speci*fier/determination, u^o_n. That is, u_n is a "**N**atural" **n**umber '"coefficient"' of/for u^o_n, specifying the *count* of **units** of the **n**th category, in a given ***categorial progression***, that are present in the given context. That -- **now** '«***monad***»-*ic*', **no longer** '*categoric*', or '"*categorial*"' -- *quant*ifier, u_n, is, in its turn, '"*qual*ified"' by that now «***monad**(s)*»-*qual*ifier, u^o_n, *symmetrically* and *mutually*, just as that *quant*ifier, u_n, *quant*ifies that *qual*ifier, u^o_n.

This u_n *quant*ifier does **not** -- *uselessly* -- *count* the ***ontological category***, '⊔$_n$', itself, as the *unit*, which would always, invariably, result in a count of just **1**, given the "idempotent" kind of addition of likes -- the 'super-amalgamative' kind of addition of likes -- that characterizes the '**un**addable'[Plato: «***a**sumbeltoi*»] ***ontological-categorial qual***ifiers of the $_N\mathbf{Q}$ axiom**s**-system of ***dialectical arithmetic***, given it axioms. Instead, the value of u_n represents the *count* of the [relative] ***base-level individual*** **units** of **kind** ⊔$_n$ that are, e.g., present **im**plicitly in the corresponding ⊔$_n$ of the current **s**tep of the presentation, with the, respectively, ***ex**plicitly* '≡' **im**plicitly '***quanto**-**qual**itative value/equation*' $1u^o_n \equiv u^o_n$ denoting a *single* such [relative] ***base-level individual*** **unit** of **kind** ⊔$_n$. With this new, $_N\underline{\mathbf{U}}$, arithmetical / ideographical language, we can now explicitly *translate* -- into mathematical, arithmetical, 'algorithmical', 'ideogramic' ***shorthand*** -- e.g., *translate* English, spoken or written, multi-vocal or multi-phonogramic, vocalizations / symbols-strings, or 'multi-phonetic utterances' / 'multi-symbol-writings', such as "**three apples**", or such as "**three pounds**", *into* and *by means of* ideogramic ***expressions*** / '***compound meta**-**meta**-**numerals***' of the **gene**ral form [i.e., of the ***algebraic*** form] $u_n u^o_n$.

Thus, if u_1 were to be assigned the "**N**atural" **number** arithmetical *quant*ifier value of **3**, and if $\underline{u}^o_1$ were to be assigned to the 'ontological *qual*ifier' "**apples**", '$\underline{u}^o_1 \looparrowleft \hspace{-2mm}\longrightarrow \underline{a} \looparrowleft \hspace{-2mm}\longrightarrow$ **apples** ', with "**apples**" also denoted by $\underline{a}$, then --

$$u_1\underline{u}^o_1 = 3 \otimes \underline{u}^o_1 = 3\underline{u}^o_1 \looparrowleft \hspace{-2mm}\longrightarrow 3\underline{a}$$ -- would stand for the English phrase "**three apples**".

Or, if u_1 were to be, *again*, assigned to the "**N**atural" number arithmetical *quant*ifier value **3**, and if $\underline{u}^o_1$ were to be assigned, instead, to the '**metrical** *qual*ifier' "**pounds**", also denoted by $\underline{p}$, then --

$$u_1\underline{u}^o_1 = 3 \otimes \underline{u}^o_1 = 3\underline{u}^o_1 = 3\underline{p}$$

-- would stand for the English phrase "**three pounds**".

Note that we still cannot, yet, within the mathematical / arithmetical facilities / confines of the $_\mathbf{N}\mathbf{U}$ language, express, e.g., English, phrases such as "**three pounds** [**of**] **apples**", in which both an '*ontological category qual*ifier', in this case, "**apples**", and a '**metrical unit** *qual*ifier', in this case, "**pounds**", as well as a '**metrical** *quant*ifier', in this case "**three**" -- all three components / '''*factors*''' -- all appear at once / all occur '''**multiplied**''' together, per our *gene*ralized concept of [*gene*rally **non**-amalgamative] '''**multiplication**'''. Such triple**+** conjunctions occur for "**state**-space"-trajectory-describing '**state**-[variable][-vector]s', e.g. --

(1), for the 'state-[variable][vector]' --

$$(m_1 \otimes dr_{1,x}(t)/dt) \otimes [[\underline{M} \otimes \underline{L}] / \underline{T}] \otimes [\underline{p}_{1,x}] \otimes [\underline{x}]$$

-- for the physical-space-model's **x**-axis directional coordinate/component for the ***momentum*** of "particle" **1** as a function of **t**ime, **t**, a thus ***dynamical* s**tate-variable, represented by **s**tate-variable ['''ontological'''] *qual*ifier $\underline{p}_{1,x}$, whose '*metrical unit quant*ifier' is --

$$(m_1 \otimes dr_{1,x}(t)/dt)$$

-- which '''"modifies''' / is **measured** in terms of the '**metrical** *unit*-/«*monad*»-*qual*ifier' of **L**ength, **[$\underline{L}$]**, say, **measured** in the *unit*-of-**measure** *qual*itative **metrical** «*monad*» of the "**inch**", ***divided*** [**/**] ***by* T**ime, **[$\underline{T}$]**, say, **measured** in the *qual*itative *unit*-of-**measure metrical** «*monad*» of the "**hour**", together forming the "compound" '**metrical** «*monad*»', or "**metrical unit**", of **V**elocity **[$\underline{V}$]** = **[$\underline{L}$ / $\underline{T}$]**, thus **measured** in *units* of "**inches per hour**", thence forming the *further*-'''compounded''' '**metrical** «*monad*»' of "**momentum**", **[$\underline{P}$]**, via **M**ass, **[$\underline{M}$]**, **measured**, say, in the *unit*-of-**measure** *qual*itative **metrical** «*monad*» of the "**pound**" --

$$[\underline{P}] = [\underline{MV}] = [\underline{ML} / \underline{T}] = [[\underline{M} \otimes \underline{L}] / \underline{T}]$$

-- thus **measured**, *in toto*, in the compound[ed] '**metrical** «*monad*»' of "**pound**-**inches per hour**", for this *classical* "phase-space" type of **s**tate-space, thus simplifying this state-vector-value to --

$$(m_1 dr_{1,x}(t)/dt)[\underline{Pp}_{1,x}\underline{x}]$$

-- or --

(2), for the "state-[variable][vector]" --

$$(r_{1,x}(t)) \otimes [\underline{L}] \otimes [\underline{r}_{1,x}] \otimes [\underline{x}]$$

-- for the **x**-axis physical-space-model's **x**-axis directional coordinate/component of the **_position_** of "particle" **1** as a function of **t**ime, **t**, whose '**metrical _quant_**ifier' is **(r$_{1,x}$(t))**, which '''*modifies*''' / is also **measured** in terms of the '**metrical** «*monad*»-*qual*ifier'' of **L**ength, **[L]**, say, **measured**, again, in the **_unit_**-of-**measure _qual_**itative **metrical** «*monad*», or **metrical _unit_**, of the "**inch**", and simplifying to --

$$(r_{1,x}(t))[\underline{L r_{1,x} x}]$$

-- or --

(3), for a *classical* "**phase**-space"-*associated* '[**c**ontrol-][parameter-]space" '[**c**ontrol-][parameter-][vector]', or, equally, for a _non_-classical "**state**-space"-*associated*, "[**c**ontrol-][parameter-]space" '[**c**ontrol-][parameter-][vector]', e.g. --

$$(m_1) \otimes [\underline{M}] \otimes [\underline{c_1}]$$

-- for the, typically-**_ass_**umed **_constant_**, or time-**_non_**-varying, **_mass_** of "particle" **1**, whose '**metrical _quant_**ifier' is **(m$_1$)**, which '''*modifies'*'' / is also **measured** in terms of the '**metrical** «*monad*»-*qual*ifier' of **M**ass, **[M]**, say, **measured**, again, in the **_unit_**-of-**measure _qual_**itative **metrical** «*monad*» of the "**pound**", and whose '[**c**ontrol-][parameter-]space' -- in this case, '**masses**-space' -- [**_non_**-physical-space-model] directional **_unit_**-vector for "particle" **1** is denoted by **[c$_1$]**. The '**c**ontrol-parameter-vector', in this case, thus simplifies to --

$$(m_1)[\underline{M c_1}].$$

All three examples, above, apply to the classical, "**phase**-space" type of "**state**-space" [including to its associated "**control**-[parameter-]space", which we call "**masses**-space"], e.g., for a [*nonlinear*] **_dynamical systems theory_** total-differential equation(s)-system[, and, typically, also, given the special propensity toward "*singularity*" inherent in "higher [than] degree [**1**]" such **_nonlinearity_**, for a "*singularity*"-entailing, and thus also **_meta_**-]*dynamical* mathematical model.

Note that each of these first two, "**phase**-space", or *classical* "**state**-space", examples actually each require _not_ three but four '''factors''', '''*speci*fiers''', 'determinors', or '''modifiers''' -- one '[metrical] *quant*ifier', "times" one '[metrical] *qual*ifier' [similar to what the $_N\underline{U}$ system _can_ provide], but also "times" one "state-variable" '*ontological-categorial qual*ifier', "times" one 'spatial-directional-vector' *qual*ifier, and so exceeds the ideographical linguistic capabilities of the $_N\underline{U}$ arithmetical/algebraical language by _not_ just one but by two kinds of additional 'arithmetical-ideographical *qual*ifier' '''factors'''/'''*speci*fiers'''/'determinors'/'''modifiers'''. Capability to express **_quant_**ifiers in '*gene*ralized-multiplicative' combination/conjunction with *both* **metrical _qual_ifier units** and **ontological** [e.g., state-variable or control-parameter 'identifier'/'''specifier''] *qual*ifier **_units_**, begins _not_ with $_N\underline{U} = {}_N\hat{\mathbb{Q}}_{QN}$, the *third* axiom_s_-system in this axiom_s_-*systems progression presentation* -- its *first* '**_dialectical_** full-*synthesis*' axiom_s_-system -- but with its **seventh** axiom_s_-system -- the *second* '**_dialectical_** full-*synthesis*' axiom_s_-system -- which we of **F**.**_E_**.**D**. denote by $_N\hat{\mathbb{Q}}_{MQN}$, or by $_N\hat{\mathbb{Q}}_{MU}$, or by $_N\underline{U}$, the '**Mu**' axiom_s_-system, wherein $_N\hat{\mathbb{Q}}_{QQ} = {}_N\hat{\mathbb{Q}}_M = {}_N\underline{M}$ denotes the **_fourth dialectical arithmetic_** axiom_s_-system, the axiom_s_-system for '**_unquant_**ifiable **M**etrical *qual*ifiers'.

All three of the symbols $_N\widehat{Q}_{QN}$, $_N\widehat{Q}_U$, and $_N\underline{U}$, stand for a new "first-order" axiom_s-system which *'complexes-together'* or which *'''compounds'''*, or which *'uni[t-i]fies'* -- which constitutes a *dialectical synthesis* of -- the axiom_s-system $_N\underline{Q}$, and of its space, $_N\underline{Q}$, i.e., of its *units*, $\overline{\underline{\Psi}}_n$, *become* $\underline{u}^o_n$, and the axiom_s-system $\underline{N}$, and of its space, N, i.e., of its *elements*, n, *become* u_n.

As we have seen above, the u_n component of the new, *'''compound unit'''*, stands for a «*monad*[*s*]»[-*ic-level*]-*quant*ifier, or *'base-level units-quant*ifier', the *count*[*or*], for the *cardinal number* -- the N *of* N number -- for the $\underline{u}^o_n$ *units* / «*monads*» *present*, and that this $\underline{u}^o_n$ component stands for a *'unit-qual*ifier', or [*relative*] *base-level* «*monad*[*s*]»[-*ic*]-*qual*ifier, *no longer* for a *'category-as-unit-level qual*ifier', as $\overline{\underline{\Psi}}_n$ did.

Thus, the compound *'meta-meta-numerals'* of the $_N\underline{U}$ *ex*plicit-*dialectical arithmetic* are *'''complex*[*es of /*] *unities'''* *of* the *numerals* of the $\underline{N}$ *im*plicitly-*dialectical arithmetic* *and* *of / together with* the *'meta-numerals'* of the $_N\underline{Q}$ *ex*plicitly-*dialectical arithmetic*. That is, the transition *from* the n, and *from* the $\overline{\underline{\Psi}}_n$, onward / upward *to* the $u_n\underline{u}^o_n$, or *to* their *dynamical* versions, e.g., *to* their *'''population dynamics'''* *versions*, such as $u_n(\tau)\underline{u}^o_n$, *forces* that which was, in $_N\underline{Q}$, the merely *im*plicit *presence* of the *base-level* [*sub-*]*units*, or [*sub-*]«*monads*», of each $\overline{\underline{\Psi}}_n$ *'category-level-as-unit-level ontological qual*ifier', or of each $\overline{\underline{\Psi}}_n$ '«*arithmos*»-*qua-unit*/«*monad*» *qual*ifier', to the surface -- into *ex*plicit, counted -- *quant*ified -- recognition. It so *forces* because it is *not* meaningful or useful / 'use-valu[e-]able', in *dialectical modeling*, to count more than one, supposedly "**identical**" copy of "the same" ontological *category*, e.g., to have $3\underline{u}^o_1$ stand for the 'co-*presence'* of three "**identical**" copies of the entire *category* "**apples**", or of $\underline{a}$, itself -- $3\underline{a} \equiv \underline{a}$ -- rather than denoting the 'co-*presence'* of three [*similar*, but *not* even possibly "**absolutely identical**"] «*monads*», *units*, or *individuals*, e.g., that presently make up the entire extant *'''population'''* of that *category*, in this case, the *'''population'''* of three *similar individual apples*.

Summary Proposition Regarding the Scientific Utility of *'Generic ORDinality'* vs. that of *'Generic CARDinality'*.

The abstract[ed] property / characteristic / determination of *'gene*ric *qual*itative *ord*inality' has far greater scientific, empirically-valid, meaningful *universality* than does the abstract[ed] property / characteristic / determination of *'gene*ric *qual*itative *card*inality'.

That is, the *'ord*inal *qual*ifier categories' of *'first-ness'*, *'second-ness'*, and *'third-ness'*, etc., have a *gene*ric, *universal*, empirical meaning, especially regarding "chronological *ord*er", as well as 'systematic *ord*er' [i.e., 'classificatory *ord*er'; 'taxonomic *ord*er'], that the *'card*inal *qual*ifier categories' of *'one-ness'*, *'two-ness'*, and *'three-ness'*, etc., do *not*.

The *presumption* of the **universality** of *card*inalities -- the fixed idea that abstract, *'unqual*ified *card*inal *quant*ities', and their *'card*inal *qual*ities', have the same *gene*ric, core, *trans-quant*itative, *qual*itative meaning in *all speci*al contexts, *universally* -- engenders the Ancient/Modern *pseudo*-science of "**numerology**".

On the contrary, the inductively grounded *expectation* of the universality of **ord**inalities, of the ubiquity of ceaseless, diverse empirical ***instancings*** of **ord**ered, consecutive sequencings, whether in the *dia*chronic domain of "chronological **ord**er", in the *syn*chronic domain of 'taxonological **ord**er' [the **ord**er that moves from the more *speci*fic category to the **next** more *gene*ral category, or from the more *gene*ral category to the **next** more *speci*fic category], or in both, is the very basis of modern, empirically-disciplined scientific theory, as well as of '*dialogical dialectic*' probing of the modern '*Human Phenome*', and of **dialectical** **science**, which the **algebra** of the $_N\mathbf{Q}$ **dialectical** **arithmetic**, together with its '*organonic algebraic method*', provide an '''«*organon*»''', an *algorithmic*-«*praxis*» '''*cognitive tool kit*''', for both the Marxian, **dialectical** '''*method of presentation*''', and the Marxian, **dialectical** '''*method of discovery*'''.

The radical root of the ultimate modern abstraction and radical simplification of human natural language/cognition, in the form of an "artificial", ideographical, 'algorithmical', arithmetical written language of apparently *"pure"* -- 'metrically-**unqual**ified' and 'ontologically-**unqual**ified' -- **card**inal **quant**ifiers, is a largely-**un**conscious '*psychohistorical* **construct**' named, by Marx, *"The Elementary Form of* [*Commodity*] *Value"*, the **foundation** -- the '$\boxed{\mathbb{Q}}_{\llcorner 1}$-assignable' '«*arché*»-*thesis*' -- of the entire vast edifice of Marx's immanent critique of capitalist political-economics.

Moreover, that **un**conscious paradigm also resides, **un**critically, at the heart of the ***historically-specifically*** Modern '*Human Phenome*', as the **foundation** of '*The Modern Ideology*', as of '*The Modern* «*mentalité*»' *entire*.

By themselves, in their *gene*ric, **un**interrupted form -- or, more exactly stated, in their *minimally*-interpreted form -- the $_N\mathbf{Q} \equiv \left\{ \boxed{\mathbb{Q}}_{\llcorner n} \right\}$ are merely the sequence / progression of abstract '*qual*itative **ord**inality', or of the **universal** '*ord*inal *qual*ities' -- '*first-ness*', '*second-ness*', and '*third-ness*', etc.

For any *speci*fic, particular **dialectical modeling** application of the $\left\{ \boxed{\mathbb{Q}}_{\llcorner n} \right\}$, these *gene*ric *qual*ifiers must be, precisely, '**de**-*gene*ricized' -- must be '''*speci*fied''', or '*speci*ficitized' -- by the modeler: "interpreted" as, "assigned" to, "applied" to the *speci*al case that is to be modeled, the *speci*al case from out of that *gene*ric, *gene*ralized **dialectic** that is encoded by the $\left\{ \boxed{\mathbb{Q}}_{\llcorner n} \right\}$, and that is to be modeled, using '*speci*fitized', '''**algebraicized**''' versions of the *gene*ric $\left\{ \boxed{\mathbb{Q}}_{\llcorner n} \right\}$.

The 'human-evolutionarily' **un**avoidable, historically-necessary, one-sidedly *"quant*itative" «*mentalité*» of **Modern**, **capitalist humanity** -- not just '''**The *German* Ideology**''', but '**The *Modern* Ideology**' *entire* -- has yielded marke*d*, market *advantages* for recent humanity, throughout the '''ascendance phase''' of the capitals-system, human-societal reproductive fitness *advantages* over and above the *contra*-*advantages* of those still mired in its predecessor human «*mentalités*».

But that one-sidedly *"quant*itative" «*mentalité*» -- the «*mentalité*» of the '**Capital**-*Value Mind*', and of its "**standard**", **"N**atural**"** arithmetic -- is also ***debilitating*** for modern humanity, especially in the present, '''descendance phase''' of the capitals-system.

It is ***debilitating***, for example -- in just one particular way from out of many particular ways -- *scientifically*, and in the *engineering* applications of *the sciences* that help directly to grow *the human social forces of production*, especially with regard to the -- supposedly "impossible" -- "analytical", "closed-form" solution of the "laws"-of-nature-formulating, *nonlinear* "partial" and "total" *integrodifferential equations*, and with regard to the problem of the division-by-zero *"singularities"* that abound *especially* for such *nonlinear* equations, and/or for their solution-functions.

Eva Brann, the translator, into English, of Jacob Klein's magisterial work, entitled **_Greek Mathematical Thought and the Origins of Algebra_**, expressed the matter in this way --

"I have omitted here, as too complex for brief exposition, a third, more directly **global interpretation of _the modern condition_**, which is central to _Greek Mathematical Thought and the Origin of Algebra_."

"It is an understanding of **the basic rupture between _antiquity_ and _modernity_**, of _the **great revolution**_ of the West, as brought about by, **or at least paradigmatically displayed in**, the introduction of algebra."

"Algebra works with **_quantities abstracted from concrete collections_** (such as were betokened by the Greek _arithmoi_), with "general," essentially symbolical "numbers," such as the variables x, y, z or the constants a, b, c."

"These letters are symbols of a peculiar sort: they _represent_ neither a concrete thing nor a determinate concept, but rather _present_ themselves as the object of calculation -- a mere object, an indeterminate entity."

"Klein saw algebraic problem-solving procedures, so effective precisely because **so _contentlessly formal_**, as **emblematic of the _modern_ _rage_ for** that second-order, **deliberately _denatured thinking_** which **dominates as much of our lives** as is method-ridden."

"The **human consequences** of this **symbolic conceptuality** are great."

Long ago, Karl Marx revealed the _'''**psycho**historical'''_ **secret** of that _'**qual**itative-**content**-**emptied**'_, **qual**itative-**nature**-_'''**de**naturing thinking'''_, of that _'**purely-quantitative**-symbols based_, **unqualified**' _'''**symbolic conceptuality**'''_, in the very **point-of-departure** of his vast and critical theory of **globalized human modernity**, in the form of that **foundational-modern**, [[**Commodity**-]**Capital**-]**_Value category_** that he named _**"The Elementary Form of** [**Commodity**] **Value"**_, as in the following crucial passage from **_Capital_**, Vol. **I**, Part **I**, Ch. **I**., Sec. **3**., 'sub[1]-sec.' **A**., 'sub[2]-sec.' **2**., 'sub[3]-sec.' **a**. --

"In order to discover how the elementary expression of the value of a commodity lies hidden in the value-relation of two commodities, we must, in the first place, consider the latter entirely apart from its quantitative aspect. The usual mode of procedure is generally the reverse, and in the value-relation nothing is seen but the proportion between definite quantities of two different sorts of commodities that are considered equal to each other. _It is apt to be forgotten_ that the magnitudes of different things can be compared quantitatively, only when those magnitudes are expressed in terms of the same unit. It is only as expressions of the same unit that they are of the same denomination, and therefore commensurable[1]."

"1The few economists, amongst whom is S. Bailey, who have occupied themselves with the analysis of the form of value, have been unable to arrive at any result, first, because they confuse the form of value with value itself; and, second, **because**, **under the coarse influence of the practical bourgeois**, **they exclusively give their attention to the _quan_titative aspect of the question**. "The command of quantity ... constitutes value." ("Money and its Vicissitudes." London, 1837, p. 11. By S. Bailey.)**."** [_emphases **added**_ by A.D.].

Unconsciously inculcated by the incessant, permeating, daily and hourly human «**_praxis_**» of '''exchange-value exchange''' -- and by all else that inexorably comes with it -- the «**_praxis_**» that most characterizes **the Modern**, **Capital-Value-preoccupied epoch**, this _'''**aptness to forget**'''_ the entire **qual**itative side of "value", and of '''value_s_''' in **gene**ral, to **"forget"** '''what **_truly profits_** humanity''' -- as reflected, for but one '''local''' example, in the _'**elision of the qualifiers**'_ in our maths, is the _'''**psycho**historical'''_ **cognitive and moral malady**, and the human-**species**-**self**-**extinction threatening** conceptual and ethical **debility**, that is now at the top of the [**psycho**]**historical agenda** for the **self-healing** of Terran humanity. Its disposition will decide the outcome of our approaching _'**Meta**-**Darwinian Planetary Selection Test**'_, testing the **fitness**, for further '''[**meta**-]**evolution**''', of we, this humanity. It will determine the _'**New/Final Dark Age**'_ versus _'**First Global Renaissance**'_ outcome of that looming '''_**Test**_'''. We humans' **historic choice?** _'**Self-Heal**'_, or **_Die_!**