

Discovering Whole-Qualifier Space (\underline{wQ}) via W-Cum Space (\underline{Cw})

(OR: *What a difference including an “origin-element” makes!*)

by
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About this Brief: The purpose of this F.E.D.* “Brief #6” is to extend the **N-Cum** (\underline{Cn}) and \underline{nQ} co-discoveries by invoking an “origin” element ($\underline{C_0} = \underline{q_0}$) for their spaces, to obtain the **W-Cum** (\underline{Cw}) and \underline{wQ} spaces. Surprisingly, this *origin qualifier* is like both **0** (under $+$) and **1** (under \times) in the **Whole Numbers**! The inclusion of $\underline{C_0}$ then becomes the basis for expanding this “**Whole-Qualifier**” space to the co-discovery of “**Integer-Cumulation**” and “**Integer-Qualifier**” spaces (the topic of our next brief). Here, the *topic qualifier* $\underline{q_0}$ is seen as a unique “Boolean” **qualifier**, which may be “assigned” to represent a “topic” ontology. An appendix explores ways of quantifying the “definiteness of **qualifiers**,” with $\underline{q_0}$ being regarded as the “least definite” **qualifier**.

Overview of “Brief #5” and the “Origin” ‘Cumulum’ or qualifier

In F.E.D. Brief #5, “Discovering **Natural-Qualifier** Space (\underline{nQ}) via **N-Cum** Space (\underline{Cn})”, an exponential-type of isomorphism is used to map the **Naturals** onto **N-Cum** (\underline{Cn}), a “**Cumulation** space” of idea set-numbers. Using the qualo-differential operator on elements of this space, we “co-discovered” **Natural-Qualifier** space (\underline{nQ}) and its addition and multiplication rules, or “axioms”. This process of co-discovery answered several “early questions” often asked by both other F.E.D. readers, and by this author.

This process is now extended by asking a further question: “*Can we construct a cumulation, X , such that, for any n in \mathbf{N} : $X + \underline{C_n} = \underline{C_n}$?*”, or “*Can we construct an X , such that $X = \underline{C_n} \sim \underline{C_n}$?*” (If so, would $X = \underline{C_n} \sim \underline{C_n} = \{ \}$, the null-set? See last subsection of **Appendix A2**.)

In Brief #5 we defined $\underline{q_n}$ as the differential of the **n**th **cumulation**:

$$\underline{q_n} := \underline{\partial C_n} := \underline{C_n} \sim \underline{C_{n-1}} \Leftrightarrow \underline{C_n} = \underline{C_{n-1}} + \underline{\partial C_n} = \underline{C_{n-1}} + \underline{q_n},$$

and defined $\underline{\partial C_1} := \underline{C_1}$. This implies that, for $n = 1$, we’d have $\underline{C_1} = \underline{C_{1-1}} + \underline{\partial C_1} = \underline{C_0} + \underline{C_1}$. So, for $n = 0$, $\underline{C_0}$ (*without underline*) would be such an X . Therefore, we postulate that such an $X = \underline{C_0}$ exists, and that its differential is defined as itself (as also for $n = 1$, $\underline{\partial C_1} = \underline{C_1}$):

“Origin-Cum/Qualifier” Existence Postulate: There exists an *origin* (or *null*) **cumulation**, $\underline{C_0} := (\underline{C_1})^0$, or *origin* (or *null*) **qualifier**, $\underline{q_0} := \underline{\partial C_0} := \underline{C_0}$, less “definite” than $\underline{q_1}$, and such that $\underline{C_0} + \underline{C_n} = \underline{C_n}$, for every n in \mathbf{N} .

Using our linear $\underline{\partial}$ on $\underline{C_0} + \underline{C_n} = \underline{C_n}$, we see that it follows immediately that $\underline{\partial C_0} + \underline{\partial C_n} = \underline{\partial C_n}$, or that $\underline{q_0} + \underline{q_n} = \underline{q_n}$, since $\underline{q_k} := \underline{\partial C_k}$ for any k in $\mathbf{N} \cup \{0\}$. Thus, our implied addition of **qualifiers** now allows another “amalgamated sum” besides $\underline{q_n} + \underline{q_n} = \underline{q_n}$, namely $\underline{q_n} + \underline{q_0} = \underline{q_n} = \underline{q_0} + \underline{q_n}$.

We now can expand both **N-Cum** (\underline{Cn}) and \underline{nQ} by appending this new element to each, just as is done when expanding the “**Natural Numbers**” to the “**Whole Numbers**”:

In **Quantitative** space, The **Whole Numbers** := $\mathbf{W} := \{0\} \cup \mathbf{N} = \{0, 1, 2, 3, \dots\}$;

In **Cumulation** space, The **W-Cumulations** := $\underline{\mathbf{C}}_{\mathbf{W}} := \{\underline{\mathbf{C}}_0\} \cup \underline{\mathbf{C}}_{\mathbf{N}} = \{\underline{\mathbf{C}}_0, \underline{\mathbf{C}}_1, \underline{\mathbf{C}}_2, \underline{\mathbf{C}}_3, \dots\}$;

In **Qualifier** space, The **Whole Qualifiers** := $\underline{\mathbf{w}}\underline{\mathbf{Q}} := \{\underline{\mathbf{q}}_0\} \cup \underline{\mathbf{N}}\underline{\mathbf{Q}} = \{\underline{\mathbf{q}}_0, \underline{\mathbf{q}}_1, \underline{\mathbf{q}}_2, \underline{\mathbf{q}}_3, \dots\}$.

We now study the interaction of the null-Cumulation \mathbf{C}_0 and the null-qualifier \mathbf{q}_0 with other elements in their respective spaces, under their respective addition and multiplication operations. Figure 1 illustrates the “appending” of this element, $\mathbf{C}_0 = \mathbf{q}_0$, to these spaces.

We first extend our isomorphism (used in Brief #5) to $\underline{\mathbf{C}}_{\mathbf{W}}$: $\mathbf{W} \rightarrow \underline{\mathbf{C}}_{\mathbf{W}}$, where $\text{exQ}(\mathbf{w}) := (\underline{\mathbf{C}}_1)^{\mathbf{w}} := \underline{\mathbf{C}}_{\mathbf{w}}$ for all \mathbf{w} in \mathbf{W} , and where $\text{exQ}(0) := (\underline{\mathbf{C}}_1)^0 := \underline{\mathbf{C}}_0$.

We then observe that \mathbf{C}_0 under the extended “Cum \times ” rule behaves as $\mathbf{0}$ does under $+$ in the **Wholes**:

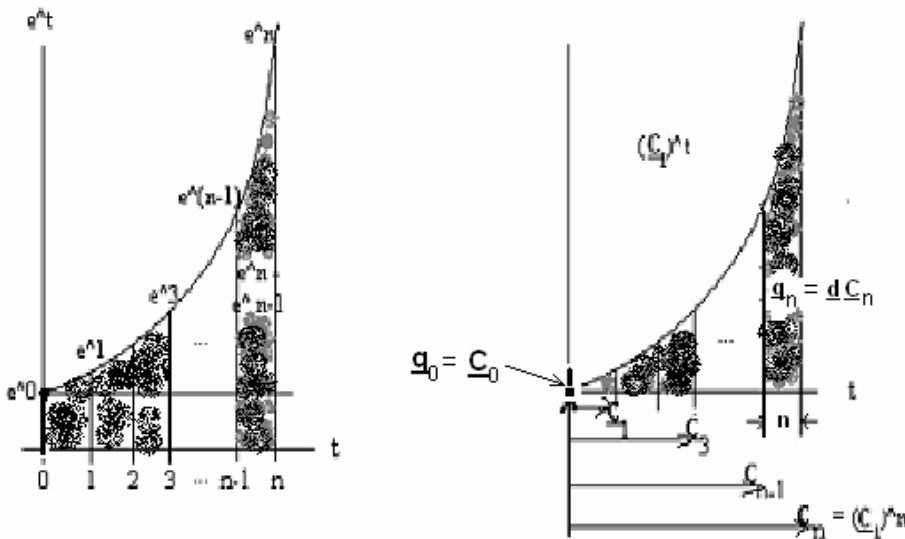
$$\text{exQ}(\mathbf{w}+\mathbf{0}) = (\underline{\mathbf{C}}_1)^{\mathbf{w}} \times (\underline{\mathbf{C}}_1)^0 = \underline{\mathbf{C}}_{\mathbf{w}} \times \underline{\mathbf{C}}_0 = \underline{\mathbf{C}}_{\mathbf{w}+\mathbf{0}} = \underline{\mathbf{C}}_{\mathbf{w}}, \text{ for all } \mathbf{w} \text{ in } \mathbf{W}, \&$$

$$\text{for all } \underline{\mathbf{C}}_{\mathbf{w}} \text{ in } \underline{\mathbf{C}}_{\mathbf{W}}.$$

Thus, $\mathbf{C}_0 = \text{id}(\text{Cum } \times)$, or the “origin-cumulum”, is the multiplicative identity for $\underline{\mathbf{C}}_{\mathbf{W}}$!

\mathbf{C}_0 as a “Line/Point Cumulation”: In Brief #5, each $\underline{\mathbf{C}}_n := \text{exQ}(\mathbf{n}) := (\underline{\mathbf{C}}_1)^n$ was viewed as a “cumulative qualitative area” analogous to “quantitative area under e^t on the t interval $[0, n]$ ”, or $\underline{\mathbf{C}}_n$ was viewed simply as the image of the point \mathbf{n} of \mathbf{N} , just as we now assume \mathbf{C}_0 to be the image of point $\mathbf{0}$ of \mathbf{W} . Yet, in Figure 1 we depict the Cumulation \mathbf{C}_0 as a “1-D line-segment, or 0-D point” rather than as a “2-D area”. Why? Briefly: To answer, we might instead think of each $\underline{\mathbf{C}}_n$ as the image of [part-open] interval $(0, n]$ of the Reals, rather than of the point \mathbf{n} . In this way, for $\mathbf{n} > 0$, $\underline{\mathbf{C}}_n$ is a 2-D image of a 1-D interval $(0, n]$, but \mathbf{C}_0 is a “1-D line-segment” image of a “0-D closed point interval”: $[0, 0]$. In our next brief, our *origin Cum*, \mathbf{C}_0 , will be used as a pivot “point” or “line”, one that is also the origin to another Cum space, viz., the “opposite” or “complementary” Cums to the Cums of $\underline{\mathbf{C}}_{\mathbf{N}}$. The reader may also regard this “point/line” aspect as a temporary reason why \mathbf{C}_0 is not underlined while the other $\underline{\mathbf{C}}_n$ (for $\mathbf{n} > 0$) are underlined; the real reason is given in the Note on $\mathbf{C}_0 = \mathbf{q}_0$ not being underlined (below).

Figure 1: Appending of $\mathbf{C}_0 = \mathbf{q}_0$ to form the **W-Cum** ($\underline{\mathbf{C}}_{\mathbf{W}}$) and $\underline{\mathbf{w}}\underline{\mathbf{Q}}$ spaces.



Next, we examine \mathbf{C}_0 's behavior under $\underline{\mathbf{C}}_w$'s extended +:

$$\underline{\mathbf{C}}_w + \mathbf{C}_0 = \underline{\mathbf{C}}_{\max\{w,0\}} = \underline{\mathbf{C}}_w, \text{ for all } w \text{ in } \mathbf{W}, \text{ i.e., for all } \underline{\mathbf{C}}_w \text{ in } \underline{\mathbf{C}}_w.$$

Thus, $\mathbf{C}_0 = \text{id}(\text{Cum } +)$, or the “origin-cumulum” is also the additive identity for $\underline{\mathbf{C}}_w$!

That “ $\text{id}(\text{Cum } \times) = \mathbf{C}_0 = \text{id}(\text{Cum } +)$ ” is an amazing result, because such a result is *impossible* in the mathematical system that is called an “algebraic field”, such as the **Q**uotient numbers (Fractions) or the **R**eal numbers, or even the **C**omplex numbers for that matter! As proven here, in **Appendix A1**, such a result is possible if and only if ‘ $\mathbf{A} + \mathbf{A} = \mathbf{A}$ for all \mathbf{A} in \mathbf{S} , an *associative, distributive* **S**ystem with $\text{id}(+)$, $\text{id}(\times)$, and $[-\text{id}(\times)]$ ’, as is the case in both the space of the **W-C**ums ($\underline{\mathbf{C}}_w$), and in the ${}_w\mathbf{Q}$ space.

We now confirm that “ $\text{id}(\times) = \mathbf{q}_0 = \text{id}(+)$ ” for the extended + and \times on the ${}_w\mathbf{Q}$ space elements:

$$\mathbf{q}_0 + \underline{\mathbf{q}}_w = \underline{\mathbf{q}}_w, \text{ for every } w \text{ in } \mathbf{W}, \text{ and for every } \underline{\mathbf{q}}_w \text{ in } {}_w\mathbf{Q} \text{ (shown earlier),}$$

and, for all w in \mathbf{W} , and for all $\underline{\mathbf{q}}_w$ in ${}_w\mathbf{Q}$, we have both products [commutative in this case]:

$$\underline{\mathbf{q}}_w \times \mathbf{q}_0 = \mathbf{q}_0 + \underline{\mathbf{q}}_{0+w} = \mathbf{q}_0 + \underline{\mathbf{q}}_w = \underline{\mathbf{q}}_w$$

and

$$\mathbf{q}_0 \times \underline{\mathbf{q}}_w = \underline{\mathbf{q}}_w + \underline{\mathbf{q}}_{w+0} = \underline{\mathbf{q}}_w + \underline{\mathbf{q}}_w = \underline{\mathbf{q}}_w.$$

Thus, $\text{id}(\times) = \mathbf{q}_0 = \text{id}(+)$, or the “origin qualifier” is both additive & multiplicative identity element in ${}_w\mathbf{Q}$!

Note on “ $\underline{\mathbf{OQ}}_w$ ” space: It can also be shown (as is done in the appendices of Brief #4) that \mathbf{q}_0 is also the *additive* and *multiplicative identity element* in all of **W**-based **O**pen **Q**ualifier space, “ $\underline{\mathbf{OQ}}_w$ ”, the set or space of all finite sums (and products) of elements from **W**hole-numbers **q**ualifier space (${}_w\mathbf{Q}$). Furthermore, it can be shown that because of the behavior of its only “amalgamative sums” with its otherwise “*non*-amalgamative sums”, the + operation is “associative” in $\underline{\mathbf{OQ}}_w$, but \times is not. Mathematically, this says that $\underline{\mathbf{OQ}}_w$ is “closed” under + & \times , and that $\langle \underline{\mathbf{OQ}}_w, + \rangle$ is a “commutative monoid.” Although $\underline{\mathbf{OQ}}_w$ is operationally “closed” for the + & \times operations, we regard $\underline{\mathbf{OQ}}_w$ as “open” in another sense -- “open” to countless possible “interpretations” of any product or sum in its many modeling applications!

Note on $\mathbf{C}_0 = \mathbf{q}_0$ not being underlined: The reader may have appropriately asked, “Why isn’t either \mathbf{C}_0 or \mathbf{q}_0 underlined, as are the other **C**umulations and **q**ualifiers?” Underlining of **C**umulations and **q**ualifiers indicates their “*contra*-Boolean” nature, i.e., that $\underline{\mathbf{q}}_n \times \underline{\mathbf{q}}_n \neq \underline{\mathbf{q}}_n$ for all n of \mathbf{N} . However, for \mathbf{C}_0 , and for \mathbf{q}_0 , this not the case, since $\mathbf{C}_0 \times \mathbf{C}_0 = \mathbf{C}_{0+0} = \mathbf{C}_0$, and $\mathbf{q}_0 \times \mathbf{q}_0 = \mathbf{q}_0 + \mathbf{q}_{0+0} = \mathbf{q}_0 + \mathbf{q}_0 = \mathbf{q}_0$. So, \mathbf{C}_0 and \mathbf{q}_0 are “Boolean”: $\mathbf{C}_0^2 = \mathbf{C}_0$ and $\mathbf{q}_0^2 = \mathbf{q}_0$. Hence neither is underlined.

q₀: Its Meaning, Interpretation, Name; Interpreting $\mathbf{q}_n + \mathbf{q}_n = \mathbf{q}_n$ & $\mathbf{q}_n - \mathbf{q}_n = ?$

But what might \mathbf{q}_0 mean in terms of any assigned “ontology”? We again consult our quantitative analog, **zer0**. Zero is $\text{id}(+)$ for the **W**hole numbers: $n + 0 = n$ for

every n in \mathbf{N} . Zer0 represents 0 units of some “topic unit” for the topic to which its number space has been assigned, e.g., 0 units “of apple” in a number space assigned to represent apples. In essence, zer0 names the “essence or topic quality \mathbf{x} ” in a “space of \mathbf{x} s”. Actually, it may be easier to identify 0’s successor, 1, as representing, e.g., “1 apple unit” in a “space of apples”.

Similarly, with qualifiers, it may be easier to identify q_0 ’s successor, q_1 , as representing “the *first* “kind of being” (ontology) for a *genus* of apple qualities” in a “space of ‘apple’ *species* [kinds of being]”. Then, q_0 would represent “the *essence/topic* of $\mathbf{X} =$ ‘apple kinds of being’,” or *the ‘null ontology’ for $\mathbf{X} =$ apples*. Just as 0 represents 0 units of some *topic-unit* (e.g., $\mathbf{x} =$ apples), so does q_0 represent *the topic-essence* of ontology \mathbf{X} (e.g., about apples). In either case, to this writer it seems appropriate to label q_0 as the “origin qualifier”, but it also seems a terrible misnomer to use the term “null qualifier” in reference to the essential purpose of such a “noble number”!

However, q_0 *is* a “null qualifier” in that has “null effect”, under + or \times , upon any other qualifier, and may appear to be like the “null set”. In any sum, $q_0 + \underline{U} = \underline{U} + q_0 = \underline{U}$; in any product, $q_0 \times \underline{U} = \underline{U} \times q_0 = \underline{U}$, so q_0 seems to say: “Yes, I recognize yoU as being of the same topic ontology, therefore I support yoU and always let yoU be yoU in any interaction with me.” (At least I *thought* I heard q_0 whispering that to one of U!)

Because each name below sheds a different light/shade-of-meaning, this author uses/accepts all of the names below as names for ‘ q_0 ’:

origin qualifier, zeroth qualifier, topic qualifier, essence qualifier, null-qualifier, null-ontology!

Next, we observe that Zer0 is the *only* quantitative number having the property $0 + 0 = 0$, whereas *every* qualifier has that property: $q_n + q_n = q_n$. Zer0 (or any qualifier) cannot qualitatively augment itself, or “aggrandize itself”, via +. In its subtractive form: $0 - 0 = 0$, Zer0 can “give herself away” and still be her **Whole 0**-self. Is not this true for ideas? -- *Giving away an idea, one (as an “idea-creator”/“idea-interceptor”) can still hold it!* Thus, *sharing an idea, we still retain it!* Since each q_n is like an idea-set, it’s no surprise that $q_n + q_n = q_n$. This property reveals each q_n capable of producing endless copies of itself within the one amalgamative sum that it is! In essence, each q_n is “a potential infinity in a finite “‘one’”!”

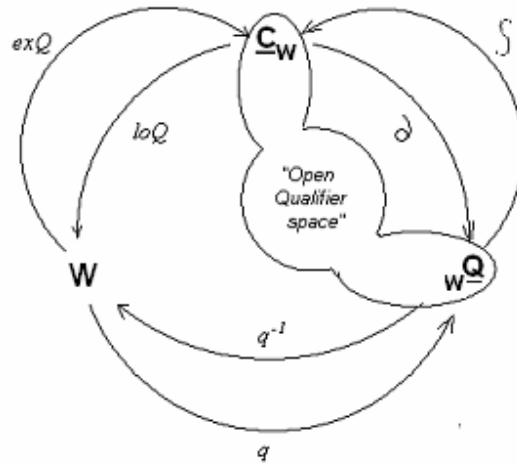
Finally, we may inquire into $w\underline{Q}$ “subtractivity”: Is $q_n - q_n = q_n$, or is $q_n - q_n = q_0$?

Or, equivalently, is $\underline{C}_n - \underline{C}_n = \underline{C}_n$, or is $\underline{C}_n + \underline{C}_{(-n)} = \underline{C}_0$? We shall address such key questions in our next brief, as we attempt to construct possible “Integer Cums”, and “Integer Qualifiers”, under their \times and + operations.

Updating Relationships

Figure 2 summarizes the functional relationships among **W**, \underline{C}_w , and $w\underline{Q}$, via function inverses $q(\)$ and $q^{-1}(\)$, $exQ(\)$ and $loQ(\)$, and $\underline{d}(\)$ and $\underline{f}(\)$, as was done in F.E.D. Brief #5. It also depicts “Open Qualifier space” as containing both the \underline{C}_w space and the $w\underline{Q}$ space, since “OQ space” is the space of all possible sums (including ‘idempotent’ or ‘single-element’ sums), and products, which arise from $w\underline{Q}$ qualifiers under addition and multiplication. (Indeed, perhaps “OQ space” really is “like a bunny rabbit’s head” because: “There are a lot of ‘sums’ between the bunny’s two ears!” Enough to constitute a monoid.)

Figure 2: Relationships of \mathbf{W} , $\underline{\mathbf{C}}_{\mathbf{W}}$, and $\mathbf{w}\underline{\mathbf{Q}}$ via $\mathbf{q}(\cdot)$, $\mathbf{q}^{-1}(\cdot)$, $\mathbf{exQ}(\cdot)$, $\mathbf{loQ}(\cdot)$, $\partial(\cdot)$, and $\mathbb{I}(\cdot)$.



Appendix A1, herein, proves that, in an associative and distributive algebraic System \mathbf{S} , with $\mathbf{id}(+)$, $\mathbf{id}(\times)$, and $[\mathbf{-id}(\times)]$, $\mathbf{A} + \mathbf{A} = \mathbf{A}$ (for every \mathbf{A} in \mathbf{S}), implies that $\mathbf{id}(\times) = \mathbf{id}(+)$, and *vice versa*. In **Appendix A2**, also herein, we speculate about the nature of --

$$\underline{\mathbf{C}}_n := (\underline{\mathbf{C}}_1)^n := \underline{\mathbf{C}}_1 \times \underline{\mathbf{C}}_1 \times \dots \times \underline{\mathbf{C}}_1 \text{ (n times)}$$

-- as a number under $\underline{\mathbf{Cum}} \times$ multiplication, including about the nature of --

$$\{\underline{\mathbf{C}}_n\} := \{\underline{\mathbf{C}}_1\}^n = \underline{\mathbf{C}}_1 \{\times\} \underline{\mathbf{C}}_1 \{\times\} \dots \{\times\} \underline{\mathbf{C}}_1 \text{ (n "set-crosses", forming sets of "ordered n-tuples")}$$

This construct has implications for the origin \mathbf{q} ualifier, and also suggests ways to begin quantifying the "definiteness of \mathbf{q} ualifiers," with \mathbf{q}_0 being regarded as the "least definite" \mathbf{q} ualifier.

Summary

By simply "appending" a "zer0-th" element ($\mathbf{C}_0 = \mathbf{q}_0$), to the $\mathbf{N}\text{-}\underline{\mathbf{Cum}}(\underline{\mathbf{C}}_N)$ and $\mathbf{N}\underline{\mathbf{Q}}$ spaces, each is extended (along with their \times and $+$ operations) to obtain the $\mathbf{W}\text{-}\underline{\mathbf{Cum}}(\underline{\mathbf{C}}_{\mathbf{W}})$ and $\mathbf{w}\underline{\mathbf{Q}}$ spaces. In " $\underline{\mathbf{O}}\mathbf{pen}\ \mathbf{W}\mathbf{hole}\text{-}\underline{\mathbf{Q}}\mathbf{ualifier}$ " space, $\underline{\mathbf{OQ}}_{\mathbf{W}}$, this origin element is shown to behave both similarly to the way that quantitative "0" (under $+$) does, and also similarly to the way that quantitative "1" (under \times) does, in $\mathbf{W}\mathbf{hole}$ number space. Thus, \mathbf{q}_0 is a unique "Boolean \mathbf{q} ualifier", which serves as the "topic/essence \mathbf{q} ualifier" for ontologies represented by, or assigned to, the $\mathbf{w}\underline{\mathbf{Q}}$.

Next, $\mathbf{C}_0 = \mathbf{q}_0$ becomes the basis for expanding these " $\mathbf{W}\mathbf{hole}\text{-}\underline{\mathbf{Q}}\mathbf{ualifier}$ " spaces, and for co-discovering "Integer- $\underline{\mathbf{Cum}}$ ulation space" and "Integer- $\underline{\mathbf{Q}}\mathbf{ualifier}$ space" (the topics of our next brief), answering questions such as: "What is $\underline{\mathbf{C}}_{(-n)} + \underline{\mathbf{C}}_n$?"

And I first thought that we weren't adding much to $\underline{\mathbf{Q}}\mathbf{ualifier}$ spaces by adding \mathbf{q}_0 .

Was I ever mistaken!

-- Joy-to-You (July, 2012)

Appendix A1 -- Proof that 'id(x) = id(+)' in a System S ⇔

'A + A = A' for any A in S

Let $\langle S, \times, + \rangle$ be an associative algebraic system wherein \times distributes over $+$, and wherein an $\text{id}(+)$, $\text{id}(x)$, and an $[-\text{id}(x)]$ exist, such that $-\text{id}(x)$ is an additive *inverse* for $\text{id}(x)$, the multiplicative *identity* element in S . The following then holds:

Theorem: $\text{id}(x) = \text{id}(+) \Leftrightarrow A + A = A$ for any A in S .

■ **Proof for:** $\text{id}(x) = \text{id}(+) \Rightarrow A + A = A$ for any A in S .

The existence and definition of $\text{id}(+)$ imply that, *in particular*, $\text{id}(+) + \text{id}(+) = \text{id}(+)$, for $A = \text{id}(+)$ in S .

Then, for every A in S , we have, given that $\text{id}(x) = \text{id}(+)$:

$$\begin{aligned} A &= A \times [\text{id}(x)] \quad [\text{by definition of } \text{id}(x)] &= \\ A \times [\text{id}(+)] & \quad [\text{substituting } \text{id}(+) \text{ for } \text{id}(x)] &= \\ A \times [\text{id}(+) + \text{id}(+)] & \quad [\text{by definition of } \text{id}(+)] &= \\ A \times [\text{id}(+)] + A \times [\text{id}(+)] & \quad [\text{given 'distributivity' of } \times \text{ over } +] &= \\ A \times [\text{id}(x)] + A \times [\text{id}(x)] & \quad [\text{substituting } \text{id}(x) \text{ for } \text{id}(+)] &= \quad [\text{by definition of } \text{id}(x)]: \\ A + A &= A \quad [\text{by 'transitivity'}]. \quad \text{Q.E.D.} \end{aligned}$$

Proof for: $A + A = A$ for any A in S [given $-\text{id}(x) \Rightarrow \text{id}(x) = \text{id}(+)$].

For the special case $A = \text{id}(x)$, the given general case $A + A = A$ implies that:

$$\begin{aligned} \text{id}(x) + \text{id}(x) &= \text{id}(x), \text{ since } A + A = A \text{ too for } A = \text{id}(x) \text{ in } S, \\ \Rightarrow \{\text{id}(x) + \text{id}(x)\} + [-\text{id}(x)] &= \text{id}(x) + [-\text{id}(x)], \text{ by adding } [-\text{id}(x)] \text{ to both sides of the equation} \\ & \quad \text{given immediately above,} \\ \Rightarrow \text{id}(x) + \{\text{id}(x) + [-\text{id}(x)]\} &= \text{id}(x) + [-\text{id}(x)], \text{ by 're-associating' LH side \{sum terms\}} \\ & \quad [\text{given 'associativity'}], \\ \Rightarrow \text{id}(x) + \text{id}(+) &= \text{id}(+), \text{ given } \text{id}(x) + [-\text{id}(x)] := \text{id}(+), \\ \Rightarrow \text{id}(x) &= \text{id}(+), \text{ by definition of } \text{id}(+), \text{ applied to the} \\ & \quad \text{LH side of the equation immediately above. Q.E.D. ■} \end{aligned}$$

Appendix A2 -- "Is \underline{C}_n a 'number', a 'set', and/or a 'set-number'?"

■ **Answer:** "All of the above!" ■

In this appendix, we speculate about the nature of the "number \underline{C}_n " defined under "Cum \times " multiplication:

$$\underline{C}_n := (\underline{C}_1)^n := \underline{C}_1 \times \underline{C}_1 \times \dots \times \underline{C}_1 \quad (n \text{ times}),$$

versus the nature of the "set \underline{C}_n ", or $\{\underline{C}_n\}$, under "set-cross" ["Cartesian Product"] multiplication, ' $\{ \times \}$ ':

$$\{\underline{C}_1\}^n = \underline{C}_1 \{ \times \} \underline{C}_1 \{ \times \} \dots \{ \times \} \underline{C}_1 \quad (n \text{ "set-crosses"}).$$

Let us, for a moment at least, redefine \underline{C}_n as connoting *both* its set and its number aspects.

First, we have the iterative "differential" and "additive" forms:

$$\underline{q}_n := \underline{\partial C}_n := \underline{C}_n \sim \underline{C}_{n-1} \Leftrightarrow \underline{C}_n = \underline{C}_{n-1} + \underline{\partial C}_n = \underline{C}_{n-1} + \underline{q}_n.$$

Key role for $\underline{\partial C}_n$ and the "set-cross" product rule:

So, let's first (and rather "naturally") define the differential increment, $\underline{\partial C}_n := \underline{C}_n \sim \underline{C}_{n-1}$, as:

$$\underline{q}_n := \underline{\partial \{\underline{C}_n\}} := \underline{\partial \{\underline{C}_1\}^n} := \underline{\partial \{\underline{C}_1 \{ \times \} \underline{C}_1 \{ \times \} \dots \{ \times \} \underline{C}_1\}} \quad (n \text{ "set-cross"} \\ \text{multiplications" yielding "ordered n-tuples"}).$$

Then let's define the entire "set-number", \underline{C}_n , as:

$$\underline{C}_n := \underline{C}_{n-1} \text{ union } \underline{\partial \{\underline{C}_1\}^n} := \underline{C}_{n-1} \{ + \} \underline{\partial \{\underline{C}_n\}} := \underline{C}_{n-1} \{ + \} \underline{\partial \{\underline{C}_1\}^n}.$$

This "construct" is chosen because it uses the "subtractive difference equation", $\underline{\partial C}_n := \underline{C}_n \sim \underline{C}_{n-1}$, to define the differential increment, $\underline{\partial C}_n := \underline{q}_n$, in terms of the "set-cross" product, $\{\underline{C}_1\}^n$. (This process seems to impart an "automatic complexification" of the initial

Cum-ontology (\underline{C}_1) set into an “ordered n -tuple” of itself (a kind of “autokinesis”?)! Then, to define the actual “set-number”, \underline{C}_n , we “add back” the previous \underline{C}_{n-1} , thus ensuring its “subsumption” into \underline{C}_n ! So, using ‘ \dots ’ as our notation for “ordered n -tuples”, if --

$\underline{C}_1 := \{p \text{ finite 'logical elements'}\} := \{e_1, e_2, \dots, e_p\}$ with $p = p^1$ logical elements, then we obtain a set of “ordered pairs”, i.e., of ‘ordered 2 -tuples’ --

$$\begin{aligned} \{\underline{C}_1\}^2 &= \{e_1, e_2, \dots, e_p\} \times \{e_1, e_2, \dots, e_p\} = \{\langle e_1, e_1 \rangle, \langle e_1, e_2 \rangle, \langle e_1, e_3 \rangle, \dots, \langle e_1, e_p \rangle\} \\ &\quad \text{union} \\ &\quad \{\langle e_2, e_1 \rangle, \langle e_2, e_2 \rangle, \langle e_2, e_3 \rangle, \dots, \langle e_2, e_p \rangle\} \\ &\quad \text{union ... union} \\ &\quad \{\langle e_p, e_1 \rangle, \langle e_p, e_2 \rangle, \langle e_p, e_3 \rangle, \dots, \langle e_p, e_p \rangle\}. \end{aligned}$$

So, the “order” or “size” of $\{\underline{C}_1\}^2$ is that of $\#\{\{\underline{C}_1\}^2\} = p^2$.

Next:

$$\{\underline{C}_1\}^3 = \{\langle x_1, x_2, x_3 \rangle, \text{ such that each } x_k \text{ is in } \underline{C}_1 = \{e_1, e_2, \dots, e_p\}, \text{ for each } k \text{ in } \{1, 2, 3\}\},$$

so that the “order” of $\{\underline{C}_1\}^3$ is that of $\#\{\{\underline{C}_1\}^3\} = p^3$.

Or, generally:

$$\{\underline{C}_1\}^n = \{\langle x_1, x_2, x_3, \dots, x_n \rangle, \text{ such that each } x_k \text{ is in } \underline{C}_1 = \{e_1, e_2, \dots, e_p\}, \text{ each } k \text{ in } \{1, 2, 3, \dots, n\}\},$$

with $\{\underline{C}_1\}^n$ having $\#\{\{\underline{C}_1\}^n\} = p^n$ elements.

Exploring measures of individual qualifier “definiteness”: **Def()** and **log_pDef()**.

Such a construct allows us to begin quantifying the “definiteness of qualifiers”, using the “order ($\#()$)” of the differential set $\underline{q}_n := \{\underline{C}_1\}^n$. Let us re-label our function $\#()$ as **Def()**, a qualo-function from qualifier space $\{\underline{q}_n\}$ into quantitative space $\{p^n\}$:

$$\underline{\text{Def}}(\underline{q}_n) := \#(\underline{q}_n) = \#\{\{\underline{C}_1\}^n\} = p^n, \text{ or simply: } \underline{\text{Def}}(\underline{q}_n) := p^n,$$

(a quantitative measure of \underline{q}_n ’s “qualitative definiteness”).

This means that **Def()** can be used to “naturally” define the “qualitative ordering” (\dashv) of $\{\underline{q}_n\}$ in terms of the quantitative ordering ($<$) in $\{p^n\}$:

$$\underline{q}_k \dashv \underline{q}_m \Leftrightarrow \underline{\text{Def}}(\underline{q}_k) < \underline{\text{Def}}(\underline{q}_m) \Leftrightarrow p^k < p^m \Leftrightarrow k < m \text{ [given } p > 1 \text{].}$$

Under this definition, \underline{q}_0 is regarded as “least definite”, with a “definiteness of \underline{q}_0 ” = $\underline{\text{Def}}(\underline{q}_0) = \underline{\text{Def}}(\{\underline{C}_1\}^0) = p^0 = 1 < p^k$, given $p > 1$. This, in turn, implies that the “initial ontology”, \underline{q}_1 , via its “interpretation”, must consist of at least two logical elements (alternatives?; “intra-duals”?), otherwise $p = 1$, and every qualifier, \underline{q}_n , would have equal “unitary definiteness”, since $\underline{\text{Def}}(\underline{q}_n) = p^n = 1^n = 1$.

Def() is only one such quantitative measure of qualitative definiteness. But, to us (to me at least), **Def()** seems, intuitively, to give “too large a value” for this “definiteness”, as it increases exponentially, as p^n . Perhaps we desire a “more moderate”, or “linear” (in n), measure. This is easily accomplished by simply taking **log_p()** of **Def()**:

$$\log_p(\underline{\text{Def}}[\underline{q}_n]) := \log_p(p^n) = n, \text{ or as one composite “log}_p\underline{\text{Def}}() \text{” function:}$$

$$\log_p\underline{\text{Def}}(\underline{q}_n) := n.$$

Actually, the triple composite function, “**log_pDef()**” (**log_pDef()** acting after the operator $\underline{\partial}()$ acts on a \underline{C}_n) maps the set of whole Cumulations, **W-Cum**, onto **W** *isomorphically*:

$$\begin{aligned} \log_p\underline{\text{Def}}[\underline{\partial}(\underline{C}_k \times \underline{C}_m)] &= \log_p\underline{\text{Def}}[\underline{\partial}(\underline{C}_{k+m})] = \log_p\underline{\text{Def}}(\underline{q}_{k+m}) = k + m \\ &= \log_p\underline{\text{Def}}(\underline{q}_k) + \log_p\underline{\text{Def}}(\underline{q}_m) = \log_p\underline{\text{Def}}(\underline{\partial}\underline{C}_k) + \log_p\underline{\text{Def}}(\underline{\partial}\underline{C}_m), \\ &\quad \text{so,} \\ \log_p\underline{\text{Def}}(\underline{C}_k \times \underline{C}_m) &= \log_p\underline{\text{Def}}(\underline{C}_k) + \log_p\underline{\text{Def}}(\underline{C}_m). \end{aligned}$$

Thus, $\log_p \underline{\text{Def}}(\)$ or $\log_p \underline{\text{Def}}\partial(\)$ offer us interesting alternatives as quantitative measures of qualitative definiteness.

A “full circle” relationship of functional composition: $\underline{\text{Def}}(\)$ and $\log_p \underline{\text{Def}}(\)$.

One cannot help but notice that all of these compositions of functions, taken together, interconnect to form a “full circle” relationship:

$$\mathbf{k} = \log_p \underline{\text{Def}}(\mathbf{q}_k) = \log_p \underline{\text{Def}}(\partial \underline{\mathbf{C}}_k) = \log_p \underline{\text{Def}}\partial(\underline{\mathbf{C}}_k) = \log_p \underline{\text{Def}}\partial[\text{exQ}(\mathbf{k})] = \log_p \underline{\text{Def}}\partial \text{exQ}(\mathbf{k}) = \text{id}_{\mathbf{N}}(\mathbf{k}),$$

or,

$\log_p \underline{\text{Def}}\partial \text{exQ}(\) = \text{id}_{\mathbf{N}}(\)$, the identity element (on \mathbf{N}) for the “composition of functions” operation,

$$\Rightarrow \log_p \underline{\text{Def}}(\) = [\partial \text{exQ}]^{-1}(\) = [\text{exQ}]^{-1}[\partial]^{-1}(\) = \underline{\text{loQ}}(\)$$

$$\Rightarrow \log_p \underline{\text{Def}}(\) = \underline{\text{loQ}}(\)$$

(such that ‘=’ as used above denotes function[al] “equivalence” or “identity”).

This last equation is an “identity” (equivalence) by definitions of the functions, each function acting on \mathbf{wQ} space elements. It expresses two “views” (similar to the way in which Maxwell’s electromagnetic field equations do, as discussed by Thomas K. Simpson in his new book, *Newton, Maxwell, Marx*):

View 1: $\log_p \underline{\text{Def}}(\mathbf{q}_k) = \log_p(\mathbf{p}^k) = \mathbf{k}$; this mapping is from \mathbf{wQ} through $\{\mathbf{p}^k\}$ onto \mathbf{W} ;

View 2: $\underline{\text{loQ}}(\mathbf{q}_k) = \underline{\text{loQ}}(\underline{\mathbf{C}}_k) = \mathbf{k}$; this mapping is from \mathbf{wQ} through $\underline{\mathbf{C}}_{\mathbf{w}}$ onto \mathbf{W} .

Possibly an “*Astonishing Result*”

We have saved for last what may be *our most astonishing finding*. We must now interpret ‘ $\underline{\text{Def}}(\mathbf{q}_0) = \mathbf{p}^0 = \mathbf{1}$ ’ as asserting that the “null ontology”, \mathbf{q}_0 , has exactly *one* “logical element”! But, wait a minute! Isn’t \mathbf{q}_0 supposed to be like the null set, $\{\ }$, which has $\mathbf{n0}$ elements? (Note that $\log_p \underline{\text{Def}}(\mathbf{q}_0) = \mathbf{0}$, however.) Rather than hastily “re-defining” the generalized “order function”, on/at \mathbf{q}_0 , to be ‘ $\underline{\text{Def}}(\mathbf{q}_0) := \mathbf{0}$ ’, let’s consider simply *accepting its implication*:

■ *The “set-number” \mathbf{q}_0 has 1 logical element, perhaps one which is somehow “uncounted” and “unseen”!*

This would make sense if we regarded --

$$\mathbf{q}_0 := \mathbf{q}_0 + \{\underline{\mathbf{C}}_1\}^0 = \mathbf{q}_0 + \{\underline{\mathbf{C}}_1 \text{ un-crossed with itself}\} = \mathbf{q}_0 + \{\underline{\mathbf{C}}_1 \{/\} \underline{\mathbf{C}}_1\} = \mathbf{q}_0 + \{\mathbf{q}_0\} = \mathbf{q}_0$$

-- a set containing itself !?

This is the enigma we are left with: *That the origin-qualifier “set-number” may have one “unseen logical element” within it – itself!* And this makes complete sense if \mathbf{q}_0 is to “contain” the “essential idea” or “topic” of the entire assigned ontology, of which it is the “origin”. ■

-- Joy-to-You !

F.E.D.** = **Foundation *Encyclopedia Dialectica, authors of the book:

A Dialectical “Theory of Everything” –

Meta-Genealogies of the Universe and of Its Sub-Universes:

A Graphical Manifesto, Volume 0: Foundations.

Websites providing free download of **F.E.D.** “primer” texts include --

www.dialectics.org and **www.adventures-in-dialectics.org**