Discovering **N**atural-Qualifier Space ($_{\mathbf{N}}\underline{\mathbf{Q}}$) via **N**-Cum Space ($\underline{\mathbf{C}}_{\mathbf{N}}$)

(*using differential and cumulation 'qualo-operators'*)

by

Joy-to-You

About this Brief: This 'brief' (article) is meant to answer "early-questions" (about the F.E.D.***** model) often asked by readers (including by this author). In answering those questions, both reader and author are led to "co-discover" F.E.D.'s **N**atural Qualifiers ($_{\mathbf{N}}\underline{\mathbf{Q}}$) by pathway perhaps different from the one originally discovered by Dr. Seldon. The math symbols used herein are used only for precision -- it is the questions, text, and figures which guide and explain the "co-discovery".

"Early-Questions"

When this student first began studying F.E.D.'s $_{\mathbf{N}}\underline{\mathbf{Q}}$ qualifiers and their additions and multiplications, he soon asked himself the typical *early-questions*, as you also may have asked -- questions such as:

1) Why is $_{\mathbf{N}}\underline{\mathbf{Q}} := \{\underline{\mathbf{q}}_1, \underline{\mathbf{q}}_2, \underline{\mathbf{q}}_3, \dots\}$ in sequential correspondence with $\mathbf{N} := \{1, 2, 3, \dots\}$?

2) Why does $\underline{\mathbf{q}}_n + \underline{\mathbf{q}}_n = \underline{\mathbf{q}}_n$?

3) Why can't the sum $\underline{\mathbf{q}}_k + \underline{\mathbf{q}}_n$ be in $_{\mathbf{N}}\underline{\mathbf{Q}}$, when $\mathbf{k} \neq \mathbf{n}$?

4) Why is the product, $\underline{\mathbf{q}}_k \times \underline{\mathbf{q}}_n = \underline{\mathbf{q}}_n + \underline{\mathbf{q}}_{n+k}$, defined as it is, necessarily with a $\underline{\mathbf{q}}_{n+k}$ term?

5) Why is the product non-commutative?

6) Why does $(\underline{\mathbf{q}}_1)^n = \underline{\mathbf{q}}_1 + \underline{\mathbf{q}}_2 + \dots + \underline{\mathbf{q}}_n$? Was this intentional or just an elegant result?

7) "How do we know that each succeeding $\underline{\mathbf{q}}_{k+1}$ is qualitatively more definite than the previous $\underline{\mathbf{q}}_k$ when we write $\underline{\mathbf{q}}_1 \dashrightarrow \underline{\mathbf{q}}_2 \dashrightarrow \dots \dashrightarrow \underline{\mathbf{q}}_n \dashrightarrow \dots$?

We asked: "Why?" "Why? "Why?" just like a little kid.

And, our "adult-parent" kept answering, "Because it *works* this way."

"Because after much research, F.E.D. was led to define it in these ways."

"They have their reasons!" …

"**Because!**"

Our little child quit asking just long enough for his "parent" (you/us) to study and discover enough – enough to begin to have some answers for our "child".

In writing about F.E.D.'s model [pages **7-8**, **F.*E*.*D*. *Brief #3***], this author made observations (which have stayed with him) in which "$\underline{\mathbf{q}}_1$" is likened to a "quality **e**", where "quantity **e**" is the natural exponential base in "**R**eal" ("pure-quantifier") space:

a) First, $(\underline{\mathbf{q}}_1)^{n+m} = (\underline{\mathbf{q}}_1)^n \times (\underline{\mathbf{q}}_1)^m$ is an isomorphic map from **N** into **{ Cumula }**, which is analogous to "$\mathbf{exp(n)} = \mathbf{e}^n$" in the "**R**eals": $\mathbf{e}^{n+m} = \mathbf{e}^n \times \mathbf{e}^m$, and;

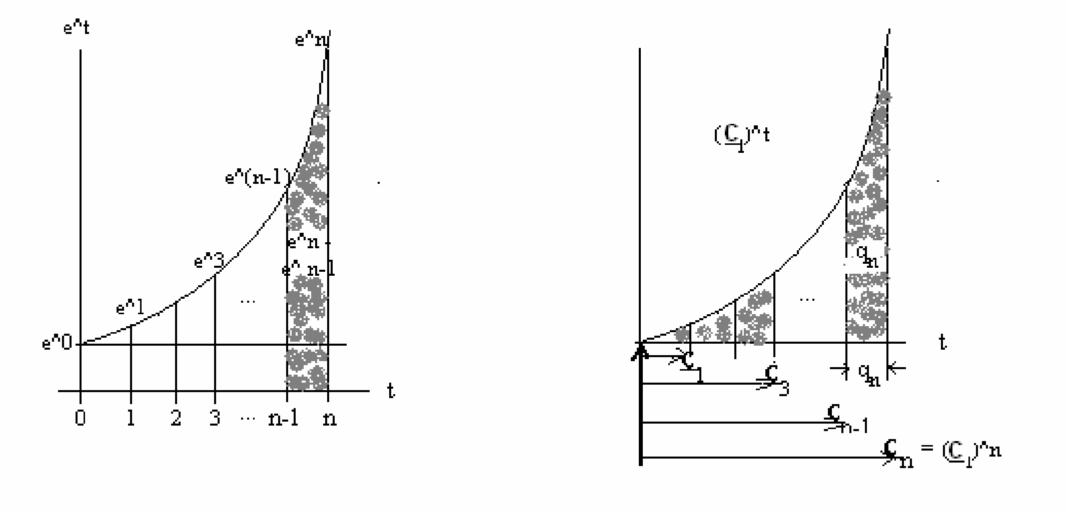
b) Second, $(\underline{\mathbf{q}}_1)^{n+m} = \underline{\mathbf{q}}_1 + \underline{\mathbf{q}}_2 + \dots + \underline{\mathbf{q}}_2$, represents a 'Cum' or '''Sum''' function … analogous to integrating the "purely-quantitative" "**exp(t)**" over the interval **[0, n]** ("epochs" **0** to **n**), where $\int \mathbf{exp(t)dt}$ [from $\mathbf{t} = \mathbf{0}$ to $\mathbf{n}$] "sums up" (is the "cumulative result" of) all historical (exponential) growth during those "epochs"!

c) For $\mathbf{t}$ in $\mathbf{[0, n]}$: in *Quantitative space*: $\mathbf{e^n} \;=\; \mathbf{e^0} + \int \mathbf{e^t dt}$; and

in ***Qualitative** space*: $(\underline{\mathbf{q}}_1)^n \;=\; \mathbf{q_0} + \Sigma\underline{\mathbf{q}}_t$.

So, not only is the "$\underline{\mathbf{q}}_1$" «***arché***»/"base" similar to the "**e**" exponential base, but on the "epoch interval" $\mathbf{[0, n]}$, "$(\underline{\mathbf{q}}_1)^n$ accumulates or sums-up" all "$\underline{\mathbf{q}}_t$" qualifiers as the integral $\int \mathbf{e^t dt}$ "sums-up" all "$\mathbf{e^t dt}$" quantities! In essence, the $(\underline{\mathbf{q}}_1)^n$ function serves as both **a)** an exponential map from $\{\mathbf{n}\}$ into $\{\underline{\mathbf{q}}_1^{\,n}\}$, and **b)** an accumulator of qualifiers, $\Sigma\underline{\mathbf{q}}_t$ – all in one "qualo-function"! Figure **1** illustrates the "$\mathbf{e^n}$ vs.$(\underline{\mathbf{q}}_1)^n$" analogy, with each $\underline{\mathbf{q}}_t$ (as $\mathbf{t}$ increases) being regarded as "qualitatively more definite" or "more refined or polished", shall we say, than is $\underline{\mathbf{q}}_{t-1}$.

Figure **1**: Analogy between Quantitative area under $\mathbf{e^t}$ vs. Qualitative elements: $(\underline{\mathbf{C}}_1)^n$ and $\underline{\mathbf{q}}_n$.



'**Idea Space**' as '**Cumulation Space**'

Our child's questioning has now turned into a mathematician's or philosopher's design question*: "If we were to design a space of 'idea-numbers', which might serve as 'container' sets of "logical elements" or sets constituting an "idea-ontology," what properties would we require of such sets or set-numbers?"*

Since we have studied the cumulation property embodied in $(\underline{\mathbf{q}}_1)^n$ and since hindsight is always perfect, we might require that those set-numbers correspond in a direct way with the "**N**atural" numbers, and that each successive set be contained in all succeeding sets:

$$\underline{\mathbf{C}}_n \text{ within } \underline{\mathbf{C}}_{n+1}, \text{ or } \underline{\mathbf{C}}_n \subset \underline{\mathbf{C}}_{n+1} \text{ [for all } \mathbf{n} \text{ in } \mathbf{N}].$$

This "contained-in" relationship reflects our intuitive wish that the "quality-set/-number" $\underline{\mathbf{C}}_n$ grow as $\mathbf{n}$ grows, reflecting greater cumulative "quality" or "knowledge" of some sort.

Next, we might ask that the first **N**atural "**1**" be mapped into the first 'acCumulation' set, $\underline{\mathbf{C}}_1$, and that this set might be used to generate subsequent 'acCumulation sets', just as "**1**" generates any $\mathbf{n}$ under repeated addition: $\mathbf{1 + 1 + \ldots + 1}$ ($\mathbf{n}$ times) $=$ $\mathbf{n}$ [for all $\mathbf{n}$ in $\mathbf{N}$]. [*Note*: We are "discovering cumulation" first.]

But since $\underline{\mathbf{C}}_k \subset \underline{\mathbf{C}}_n$ (for $\mathbf{k < n}$, where $\mathbf{k}$, $\mathbf{n}$ are in $\mathbf{N}$), then $\underline{\mathbf{C}}_k$ **union** $\underline{\mathbf{C}}_n \;=\; \underline{\mathbf{C}}_n$, this would imply a simple set-type of addition: $\underline{\mathbf{C}}_k + \underline{\mathbf{C}}_n \;=\; \underline{\mathbf{C}}_n$ when $\mathbf{k < n}$, and consequently, an idempotent addition:

$\underline{\mathbf{C}}_n + \underline{\mathbf{C}}_n \;=\; \underline{\mathbf{C}}_n$, when $\mathbf{k = n}$ (since $\underline{\mathbf{C}}_n$ **union** $\underline{\mathbf{C}}_n \;=\; \underline{\mathbf{C}}_n$).

Because set-number *addition* is "non-generative", we need a "set-number *multiplication*" under which set-number $\underline{\mathbf{C}}_1$ would generate set-number $\underline{\mathbf{C}}_n$. Corresponding to how $< 1 + >$ (**n** times) generates **n** (for all **n** in $\mathbf{N}$), so $< \underline{\mathbf{C}}_1 \times >$ (**n** times) would generate $\underline{\mathbf{C}}_1 \times \underline{\mathbf{C}}_1 \times \ldots \times \underline{\mathbf{C}}_1 = (\underline{\mathbf{C}}_1)^n := \underline{\mathbf{C}}_n$ (for all **n** in $\mathbf{N}$). This formulation also reflects our normal use of language in which we often (almost unconsciously) contend that our cumulative "knowledge grows exponentially".

These loosely-stated requirements would imply an 'exponential-$\underline{\mathbf{C}}_1$' isomorphic map **exQ(_)** from $\mathbf{N}$ to some "acCumulation space" such that '**exQ(1)**' $= \underline{\mathbf{C}}_1$, and $\mathbf{exQ(n)} = (\underline{\mathbf{C}}_1)^n := \underline{\mathbf{C}}_n$. Thus, for all **n**, **m** in $\mathbf{N}$, $(\underline{\mathbf{C}}_1)^{n+m} = (\underline{\mathbf{C}}_1)^n \times (\underline{\mathbf{C}}_1)^m$, i.e., $\mathbf{exQ(n)} := (\underline{\mathbf{C}}_1)^n := \underline{\mathbf{C}}_n$ and $\mathbf{exQ(n+m)} = \mathbf{exQ(n)} \times \mathbf{exQ(m)}$:

$$\mathbf{exQ(\,)}: \mathbf{N} \to \underline{\mathbf{C}}_{\mathbf{N}} := \{ \mathbf{exQ(n)} := \underline{\mathbf{C}}_n : \underline{\mathbf{C}}_n = (\underline{\mathbf{C}}_1)^n, \text{ for all } \mathbf{n} \text{ in } \mathbf{N} \} := \text{"}\underline{\mathbf{N}\text{-Cumulation space}}\text{"}.$$

Thus, our Cumulation or "**N-Cum**" space, $\underline{\mathbf{C}}_{\mathbf{N}}$, would have the following properties:

1) $< \underline{\mathbf{C}}_{\mathbf{N}}, \times, + >$ is isomorphic to $< \mathbf{N}, +, \mathbf{max(\,)} >$, where $\underline{\mathbf{C}}_k \times \underline{\mathbf{C}}_n := \underline{\mathbf{C}}_{k+n}$, and where

   $\underline{\mathbf{C}}_k + \underline{\mathbf{C}}_n := \underline{\mathbf{C}}_{\max\{k,n\}}$ s.t. $\mathbf{max\{k, n\}} = \mathbf{n}$ if $\mathbf{n > k}$; **k** if $\mathbf{k > n}$, and s.t. each "idea-number $\underline{\mathbf{C}}_n$" behaves as a set, representing a "container set-number" for some "cumulated set of ideas".

2) $\underline{\mathbf{C}}_n + \underline{\mathbf{C}}_n = \underline{\mathbf{C}}_n$ for each **n** in $\mathbf{N}$.

3) $\underline{\mathbf{C}}_k \;\text{—+}\; \underline{\mathbf{C}}_n \Leftrightarrow \underline{\mathbf{C}}_k \subset \underline{\mathbf{C}}_n \Leftrightarrow \mathbf{k < n}$, with the new symbol, ' —+ ', signing 'lower than' in 'total qualitative knowledge' or 'qualitative definiteness' [with '⇔' signing bi-directional implication].

4) $< \underline{\mathbf{C}}_{\mathbf{N}}$, ' —+ ' $>$ is a total order, i.e., $\underline{\mathbf{C}}_1 \;\text{—+}\; \underline{\mathbf{C}}_2 \;\text{—+}\; \underline{\mathbf{C}}_3 \;\text{—+}\; \ldots$.

Note: We recognize that increasing 'total qualitative knowledge' implies a greater capacity to refine or define whatever idea-set or "ontology" is emerging. Thus, this increased "defining" results in increased 'qualitative definiteness', as **n** increases. E.g., early man looked within the jungle's (or sky's) canopy and perceived a vaguely-understood "flat earth". Only by his increased knowledge has man recognized that such a "flat earth" is merely a small surface portion of a well-defined "oblate-spheroidal Earth", i.e., human knowledge has grown qualitatively more definite – at least in regard to these "scientific matters".

Defining $_{\mathbf{N}}\underline{\mathbf{Q}}$ using the 'Differential Qualo-Operator' $\underline{\partial}$

At this point, we begin to consider the "incremental qualifier accretions" from $\underline{\mathbf{C}}_1$ to $\underline{\mathbf{C}}_2$, from $\underline{\mathbf{C}}_2$ to $\underline{\mathbf{C}}_3$, or generally, from $\underline{\mathbf{C}}_{n-1}$ to $\underline{\mathbf{C}}_n$. We define these "qualifier increments" as '$\underline{\mathbf{C}}_n \sim \underline{\mathbf{C}}_{n-1}$', where the tilde '~' denotes a "subtractive difference" as used in set notation: '$\underline{\mathbf{C}}_n \sim \underline{\mathbf{C}}_{n-1}$' or '$\underline{\mathbf{C}}_n \setminus \underline{\mathbf{C}}_{n-1}$', to denote "all elements in set $\underline{\mathbf{C}}_n$ but not in set $\underline{\mathbf{C}}_{n-1}$".

Closely related to this "incremental" or "difference set" is the notion of a "difference operator", or more precisely, a "differential operator", that operates on **N-Cum** space "Cums". This writer recalls reading (perhaps somewhere in the F.E.D. literature, citing C. Musés) where the article claims that in new number spaces the "linear" [partial] differential $\partial(\,)$ and integral $\int(\,)$ operators might be as commonplace as the four binary operations: $+, \times, -, /$. So, we eagerly endow our Cum-space, $\underline{\mathbf{C}}_{\mathbf{N}}$, with *'qualo-'* versions of such operators:

$$< \underline{\mathbf{C}}_{\mathbf{N}}, \underline{\times}, + ; \underline{\partial}, \underline{\int} > := \underline{\mathbf{N}\text{-Cumulation space}} \textbf{ with } \text{'}\underline{\textbf{qualo-operators}}\text{'}.$$

Without hesitation, we apply our differential 'qualo-operator' $\underline{\partial}$ on any element $\underline{\mathbf{C}}_n$ of $\underline{\mathbf{C}}_{\mathbf{N}}$ to *define* $\underline{\mathbf{q}}_n$, for **n** and **k** in $\mathbf{N}$, $\mathbf{k < n}$, as:

$$\underline{\partial}(\underline{\mathbf{C}}_n) := \underline{\partial \mathbf{C}}_n\big|_{\text{[with respect to]}\,\underline{\mathbf{C}}_n} = \underline{\mathbf{C}}_n \sim \underline{\mathbf{C}}_{n\text{-}1} := \underline{\mathbf{q}}_n := \underline{\partial \mathbf{C}}_n \Leftrightarrow \underline{\mathbf{C}}_n = \underline{\mathbf{C}}_{n-1} + \underline{\partial \mathbf{C}}_n = \underline{\mathbf{C}}_{n-1} + \underline{\mathbf{q}}_n,$$

or more generally,

$$\underline{\partial \mathbf{C}}_n\big|_{\underline{\mathbf{C}}_k} = \underline{\mathbf{C}}_n \sim \underline{\mathbf{C}}_{n-k-1} := \underline{\Sigma \mathbf{q}}_t\big|_{t \text{ in } [n-k,\, n]} \Leftrightarrow \underline{\mathbf{C}}_n = \underline{\mathbf{C}}_{n-k-1} + \underline{\partial \mathbf{C}}_n\big|_{\underline{\mathbf{C}}_k} = \underline{\mathbf{C}}_{n-k-1} + \underline{\Sigma \mathbf{q}}_t\big|_{t \text{ in } [n-k,\, n]}.$$

Note: Remember that '$\underline{\mathbf{C}}_n \sim \underline{\mathbf{C}}_{n\text{-}1} := \underline{\mathbf{q}}_n$' denotes an "incremental qualitative difference", which says: "$\underline{\mathbf{C}}_n$ *without* $\underline{\mathbf{C}}_{n\text{-}1}$ defines (is) the $\underline{\mathbf{q}}_n$ qualifier".

Thus, $\underline{\partial}$: $\underline{\mathbf{C}}_{\mathbf{N}} \rightarrow \{\underline{\mathbf{q}}_n = \underline{\partial \mathbf{C}}_n\text{: } \mathbf{n} \text{ in } \mathbf{N}\} = {}_{\mathbf{N}}\underline{\mathbf{Q}}$! We have defined ("co-discovered"!) that ${}_{\mathbf{N}}\underline{\mathbf{Q}}$ is the set of 'qualo-differentials' of 'Cumulation' space elements: $\underline{\partial}[\,\underline{\mathbf{C}}_{\mathbf{N}}\,] := {}_{\mathbf{N}}\underline{\mathbf{Q}}$. The differential operator operating on $\underline{\mathbf{C}}_{\mathbf{N}}$ creates ${}_{\mathbf{N}}\underline{\mathbf{Q}}$, or ${}_{\mathbf{N}}\underline{\mathbf{Q}}$ is "$\underline{\partial}$erived from" $\underline{\mathbf{C}}_{\mathbf{N}}$, its 'Cumulation space'. This answers early-question **1**, as we have "co-discovered" ${}_{\mathbf{N}}\underline{\mathbf{Q}}$ in a new way!

Now, apply our linear 'qualo-operator' $\underline{\partial}$ to the idempotent 'Cum sum': $\underline{\mathbf{C}}_n + \underline{\mathbf{C}}_n = \underline{\mathbf{C}}_n$, to obtain: $\underline{\partial}(\underline{\mathbf{C}}_n + \underline{\mathbf{C}}_n) = \underline{\partial \mathbf{C}}_n$ '+' $\underline{\partial \mathbf{C}}_n = \underline{\partial \mathbf{C}}_n$, which says $\underline{\mathbf{q}}_n$ '+' $\underline{\mathbf{q}}_n = \underline{\mathbf{q}}_n$ and defines the addition '+' of ${}_{\mathbf{N}}\underline{\mathbf{Q}}$ elements, answering early-question **2**.

Next, assume the sum $\underline{\mathbf{q}}_k + \underline{\mathbf{q}}_n = \underline{\mathbf{q}}_m$ is in ${}_{\mathbf{N}}\underline{\mathbf{Q}}$ (assuming $\mathbf{k} < \mathbf{n}$ *and* $\mathbf{m} \neq \mathbf{n}$, for $\mathbf{m}$, $\mathbf{k}$, and $\mathbf{n}$ in $\mathbf{N}$). Then $\underline{\mathbf{q}}_m = \underline{\mathbf{q}}_k + \underline{\mathbf{q}}_n = \underline{\partial \mathbf{C}}_k + \underline{\partial \mathbf{C}}_n = \underline{\partial}(\underline{\mathbf{C}}_k + \underline{\mathbf{C}}_n) = \underline{\partial}(\underline{\mathbf{C}}_n) = \underline{\mathbf{q}}_n$, so $\underline{\mathbf{q}}_m = \underline{\mathbf{q}}_n$ or $\mathbf{m} = \mathbf{n}$, contradicting our assumption that $\mathbf{m} \neq \mathbf{n}$! Thus, by this ***reductio ad absurdum*** proof, $\underline{\mathbf{q}}_k + \underline{\mathbf{q}}_n$ cannot be in ${}_{\mathbf{N}}\underline{\mathbf{Q}}$, if $\mathbf{k} < \mathbf{n}$, answering early-question **3**.

From the relation, $\underline{\mathbf{C}}_n = \underline{\mathbf{C}}_{n\text{-}1} + \underline{\mathbf{q}}_n$, we quickly discover (proven in Appendix **A1**) that, given $\underline{\mathbf{C}}_1 = \underline{\mathbf{q}}_1$, $\underline{\mathbf{C}}_2 = \underline{\mathbf{q}}_1 + \underline{\mathbf{q}}_2, \ldots$, and that $(\underline{\mathbf{q}}_1)^n = (\underline{\mathbf{C}}_1)^n = \underline{\mathbf{C}}_n = \underline{\mathbf{q}}_1 + \ldots + \underline{\mathbf{q}}_n$, as the result for $\underline{\mathbf{C}}_{\mathbf{N}}$, as previously defined by F.E.D. for $(\underline{\mathbf{q}}_1)^n$, and answering early-question **6**. These results also lead to the following conjectures on how the multiplication $(\underline{\mathbf{q}}_k$ "×" $\underline{\mathbf{q}}_n)$ of $\underline{\mathbf{q}}$ elements might be defined.

First, we know that this "×" (possibly different from $\underline{\times}$ = 'Cumulation-Space-×') must satisfy the $(\underline{\mathbf{q}}_1)^n = \underline{\mathbf{q}}_1 + \ldots + \underline{\mathbf{q}}_n$ relation/result [for all $\mathbf{n}$ in $\mathbf{N}$].

Next, since $\underline{\mathbf{q}}_k := \underline{\partial \mathbf{C}}_k$, $\underline{\mathbf{q}}_n := \underline{\partial \mathbf{C}}_n$, and since $\underline{\mathbf{C}}_k \underline{\times} \underline{\mathbf{C}}_n := \underline{\mathbf{C}}_{n+k}$, we necessarily have $\underline{\partial}(\underline{\mathbf{C}}_k \underline{\times} \underline{\mathbf{C}}_n) = \underline{\partial}(\underline{\mathbf{C}}_{n+k}) := \underline{\mathbf{q}}_{n+k}$. Thus, we would expect/require that the "$\underline{\mathbf{q}}_{n+k}$" term be part of the defined product, $\underline{\mathbf{q}}_k \underline{\times} \underline{\mathbf{q}}_n := (\underline{\partial \mathbf{C}}_k) \underline{\times} (\underline{\partial \mathbf{C}}_n)$, answering part of early-question **4**.

Possible Multiplications on ${}_{\mathbf{N}}\underline{\mathbf{Q}}$ elements

So, perhaps the most-complex of the "simple product" definitions might be [for $\mathbf{k}$, $\mathbf{n}$ in $\mathbf{N}$]:

$$\underline{\mathbf{q}}_k \text{"×"} \underline{\mathbf{q}}_n := (\underline{\partial \mathbf{C}}_k)(\underline{\partial \mathbf{C}}_n) := (\underline{\partial \mathbf{C}}_k) + (\underline{\partial \mathbf{C}}_{n+k}) + (\underline{\partial \mathbf{C}}_n) = \underline{\mathbf{q}}_k + \underline{\mathbf{q}}_{n+k} + \underline{\mathbf{q}}_n$$

[F.E.D. calls this product-definition *"the meta-genealogical evolute product rule"*.].

So, a quick, non-exhaustive list of *possible definitions for multiplication* might be:

1) $\underline{\mathbf{q}}_k$ "×" $\underline{\mathbf{q}}_n := \underline{\mathbf{q}}_{n+k}$, commutative [F.E.D. name: *"meta-heterosis convolute product"*];
2) $\underline{\mathbf{q}}_k$ "×" $\underline{\mathbf{q}}_n := \underline{\mathbf{q}}_k + \underline{\mathbf{q}}_{n+k}$, non-commutative [F.E.D. name: *"meta-catalysis evolute product"*];
3) $\underline{\mathbf{q}}_k$ "×" $\underline{\mathbf{q}}_n := \underline{\mathbf{q}}_{n+k} + \underline{\mathbf{q}}_n$, non-commutative [F.E.D. name: *"**double**-«**aufheben**» evolute product"*];
4) $\underline{\mathbf{q}}_k$ "×" $\underline{\mathbf{q}}_n := \underline{\mathbf{q}}_k + \underline{\mathbf{q}}_{n+k} + \underline{\mathbf{q}}_n$, commutative [F.E.D. name: *"meta-genealogical evolute product"*].

Definition **1** must be ruled out immediately because it implies that:

$$(\underline{q}_1)^2 = \underline{q}_1 \text{“}\times\text{”} \underline{q}_1 = \underline{q}_{1+1} = \underline{q}_2 \neq \underline{q}_1 + \underline{q}_2 = \underline{C}_2,$$

so Definition **1** implies that $(\underline{q}_1)^2 \neq \underline{C}_2$, which denies what is required.

The other three definitions meet our $(\underline{q}_1)^n = \underline{C}_n$ criterion, as shown in Appendix **A1**.

We might also rule out Definition **4** because it is commutative, and *we have implicitly required that* “×” *be defined with emphasis on just one of the two factors, namely* $\underline{q}_k$ “×” $\underline{q}_n$:= $\underline{q}_k$ *“**of**”* $\underline{q}_n$, *where the “of” implies that the second factor* ($\underline{q}_n$) *has “more influence.”* This would suggest Definition **3**, not Definition **2**, so we might define $\underline{q}_k$ *“**of**”* $\underline{q}_n$ [for **k**, **n** in **N**] as: $\underline{q}_k$ “×” $\underline{q}_n := \underline{q}_n + \underline{q}_{k+n}$, exactly as was done in the F.E.D. model! This answers early-questions **4-5**.

<u>*Notes*</u>*:* **1)** $\underline{q}_k$ “×” $\underline{q}_n := \underline{q}_n + \underline{q}_{n+k}$ ⧧ $\underline{q}_n$ “×” $\underline{q}_k := \underline{q}_k + \underline{q}_{k+n}$, i.e., these two products are “qualitatively-unequal”, (⧧), because their first terms (their “Boolean” or “conservation” terms), $\underline{q}_n$ vs. $\underline{q}_k$, if $\mathbf{n} \neq \mathbf{k}$, differ qualitatively. **2)** *Addition* of $_{\mathbf{N}}\underline{\mathbf{Q}}$ elements (terms) ***<u>is</u>*** commutative (analogous to “set-union” being commutative). **3)** A *non*-commutative “×” multiplication was defined for the ‘qualo-differential’ elements (i.e., for all of the $\underline{q}_n$ in $_{\mathbf{N}}\underline{\mathbf{Q}}$) even though the ‘cum-<u>×</u>’ of ‘Cumulation space’ ($\underline{\mathbf{C}}_{\mathbf{N}}$) is *commutative.*

[Only early-question **7** remains to be answered: “How do we know that each succeeding $\underline{q}_{k+1}$ is qualitatively more definite than the previous $\underline{q}_k$, as implied when we write $\underline{q}_1$ ⟶+ $\underline{q}_2$ ⟶+ **…** ⟶+ $\underline{q}_n$ ⟶+ **…** ? To answer this, we appeal to the analogy between $\mathbf{e}^{\mathbf{n}}$ and $(\underline{q}_1)^n$ discussed earlier. In <u>Figure 1</u> we “see” that each increment of “$\mathbf{e}^{\mathbf{t}}\mathbf{dt}$” grows and is quantitatively larger than any previous one, as **t** grows. Although we cannot directly account for the “qualitative definiteness” of each $\underline{\partial}(\underline{q}_1)^n := \underline{q}_n$, we can “reason by analogy”, and by our “<u>*Note*</u>” on “increasing definiteness”, p. **3**. Both support our case for increasing “qualitative definiteness” of each successive qualitative-increment (qualifier), as **n** increases. Hopefully this argument answers early-question **7** *for now*, for *generic* $_{\mathbf{N}}\underline{\mathbf{Q}}$ ‘meta-numerals’, until one gets into *specific <u>models</u>*, i.e., <u>*interpretations*</u>/<u>*assignments*</u> of the $\{\underline{q}_n\}$ to *specific* ontological categories.]

Using the ‘Cumulation Operator’ ⨜

The ‘Cumulation operator’, or ‘qualo-integral’, **⨜( )**, acting on $_{\mathbf{N}}\underline{\mathbf{Q}}$ elements, produces all of <u>**N-Cum**</u>ulation space’, viz., for **n** and **k** in **N**, with **k < n**:

$$⨜: {}_{\mathbf{N}}\underline{\mathbf{Q}} \rightarrow \{\underline{C}_n : \mathbf{n} \text{ in } \mathbf{N}\} := \underline{\mathbf{C}}_{\mathbf{N}}, \text{ defined by } ⨜(\partial\underline{C}_n)\,|_{\text{on } [0,\, n]} := ⨜(\underline{q}_n)\,|_{\text{on } [0,\, n]} := \underline{C}_n,$$

or more generally,

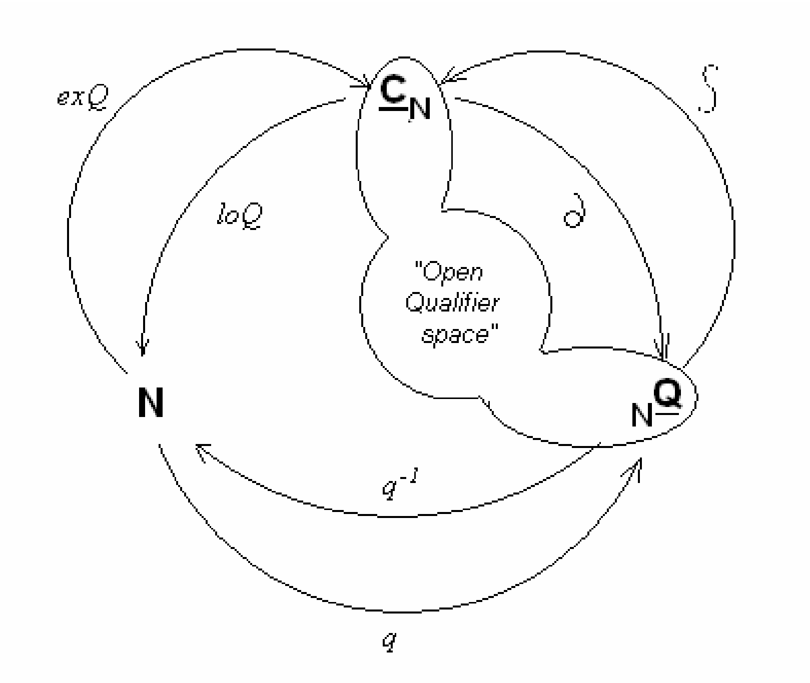
$$⨜(\partial\underline{C}_n)\,|_{\text{on } [k,\, n]} := ⨜(\partial\underline{C}_n)\,|_{\text{on } [0,\, n]} \sim ⨜(\partial\underline{C}_n)\,|_{\text{on } [0,\, k]} = \underline{C}_n \sim \underline{C}_k = \underline{\Sigma q}_t\,|_{t \text{ in } [k+1,\, n]}$$

So, we can define $\underline{\mathbf{C}}_{\mathbf{N}}$ as the set that results from the ‘qualo-integration’, or ‘qualo-cumulation’, of all elements in *‘differential space’* $_{\mathbf{N}}\underline{\mathbf{Q}}$: $⨜({}_{\mathbf{N}}\underline{\mathbf{Q}}) = \underline{\mathbf{C}}_{\mathbf{N}}$. Thus, the cumulation operator ⨜, operating on $_{\mathbf{N}}\underline{\mathbf{Q}}$, “resurrects” $\underline{\mathbf{C}}_{\mathbf{N}}$!

Thus, for any **N**atural **n**, we have: $\underline{\partial}(\mathbf{exQ(n)}) = \underline{q}_n := \mathbf{q(n)}$ or, in terms of functional composition: $\underline{\partial}\mathbf{exQ(\ )} := \mathbf{q(\ )}$, the “**q**uality map” from **N** *onto* $_{\mathbf{N}}\underline{\mathbf{Q}}$. Then, the **1**:**1**-ness of these maps allows us to define inverse mappings, $\mathbf{exQ}^{-1}(\underline{C}_n) := \mathbf{loQ}(\underline{C}_n) := \mathbf{n}$ and $\mathbf{q}^{-1}(\underline{q}_n) := \mathbf{n} := \mathbf{id_N(n)}$, so: $\mathbf{loQ}(⨜\mathbf{q(n)}) = \mathbf{n} = \mathbf{q}^{-1}(\mathbf{q(n)})$, or $\mathbf{loQ}⨜ := \mathbf{q}^{-1}\mathbf{q} := \mathbf{id_N(\ )}$, where ‘$\mathbf{id_N(\ )}$’ denotes ‘the **id**entity function’ for the elements of **N** under ‘composition of functions’.

Figure **2** summarizes functional relationships among **N**, $\underline{\mathbf{C}}_{\mathbf{N}}$, **&** ${}_{\mathbf{N}}\underline{\mathbf{Q}}$, via the paired inverse functions -- **q( )** and $\mathbf{q}^{-1}$**( )**, **exQ( )** and **loQ( )**, and **∂( )** and **∫( )**. It also depicts "**O**pen **Q**ualifier space" as containing both $\underline{\mathbf{C}}_{\mathbf{N}}$ **&** ${}_{\mathbf{N}}\underline{\mathbf{Q}}$ spaces since "**OQ** space" is the space of all possible qualifier sums (including 'idempotent sums' or 'single-element sums') arising from ${}_{\mathbf{N}}\underline{\mathbf{Q}}$ qualifiers under addition **&** multiplication. (No, "**OQ** space" is not "like a bunny rabbit's head"! Any such resemblance simply manifests this author's limited artistic skills.)

Figure **2**: Relationships of **N**, $\underline{\mathbf{C}}_{\mathbf{N}}$, and ${}_{\mathbf{N}}\underline{\mathbf{Q}}$ via **q( )**, $\mathbf{q}^{-1}$**( )**, **exQ( )**, **loQ( )**, **∂**, and **∫**.



Summary

We first learned F.E.D. theory as: The defined set of natural qualifiers, ${}_{\mathbf{N}}\underline{\mathbf{Q}}$, its properties, and its addition and multiplication operations. Then $(\underline{\mathbf{q}}_1)^{\mathbf{n}}$ was shown to be the **n**-th cumulum, and finally the set of cumula under Cum-× was shown to be isomorphic to the **N**aturals under **N** addition. Our "early-questions" about the theory led us to "answers", by "exploring-in-reverse": We first defined a "Cumulation" space $\underline{\mathbf{C}}_{\mathbf{N}}$ by an "exQ" isomorphic mapping from **N**, then, via the 'qualo-differential' operator **∂**, we made a "co-discovery" of ${}_{\mathbf{N}}\underline{\mathbf{Q}}$, its elements, its addition, and its multiplication rules!

-- *Joy-to-You* (June 2012)

F**.E***.***D***. = **Foundation** ***Encyclopedia Dialectica***, **authors of the book** ***A Dialectical "Theory of Everything"*** **–** ***Meta-Genealogies of the Universe and of Its Sub-Universes*****: A Graphical Manifesto**, **Volume 0**: **Foundations**. Websites providing free download of F.E.D. "primer" texts include --

www.dialectics.org and **www.adventures-in-dialectics.org**

Appendix A1 – Proofs that $(\underline{q}_1)^n = \underline{C}_n = \underline{q}_1 + \ldots + \underline{q}_n$ under various defined multiplication rules

We do not know *a priori* that the n-th Cumulation, $\underline{C}_n$, is equal to the n-th Cumulum, $\underline{q}_1 + \ldots + \underline{q}_n$, i.e. it must be shown that $\underline{C}_n = \underline{q}_1 + \ldots + \underline{q}_n$. By definitions, $\underline{C}_n := \underline{C}_{n-1} + \partial\underline{C}_n := \underline{C}_{n-1} + \underline{q}_n$, so if we show that $\underline{C}_k = \underline{q}_1 + \ldots + \underline{q}_k$, we have, by finite induction, that $\underline{C}_{k+1} := \underline{C}_k + \underline{q}_{k+1} = \underline{q}_1 + \ldots + \underline{q}_k + \underline{q}_{k+1}$, i.e., that the $k+1$st-Cumulation is in fact the $k+1$st-Cumulum, given the truth of "base clause" $\underline{C}_1 = \underline{q}_1$.

Our short, non-exhaustive list of *possible definitions for* $_{\mathbf{N}}\underline{\mathbf{Q}}$ *multiplication* is:

1. $\underline{q}_k$ "$\times$" $\underline{q}_n := \underline{q}_{n+k}$;
2. $\underline{q}_k$ "$\times$" $\underline{q}_n := \underline{q}_k + \underline{q}_{n+k}$;
3. $\underline{q}_k$ "$\times$" $\underline{q}_n := \underline{q}_{n+k} + \underline{q}_n$;
4. $\underline{q}_k$ "$\times$" $\underline{q}_n := \underline{q}_k + \underline{q}_{n+k} + \underline{q}_n$.

Multiplication Definition **1** was ruled out immediately because it implied that $(\underline{q}_1)^2 \neq \underline{C}_2$, which denies what is required. We shall now show that Definitions **2** through **4** all lead to $(\underline{q}_1)^n = \underline{C}_n$ for any n in $\mathbf{N}$. In each case, the "base clause" $\underline{C}_1 := \underline{q}_1$ is true by definition.

Using Definition **2**, we have that $(\underline{q}_1)^2 = \underline{q}_1$ "$\times$" $\underline{q}_1 = \underline{C}_1$ "$\times$" $\underline{q}_1 = \underline{q}_1 + \underline{q}_{1+1} = \underline{q}_1 + \underline{q}_2 := \underline{C}_2$.

Assume that this generalizes to: $(\underline{q}_1)^k = \underline{C}_{k-1} \times \underline{q}_1$, for k in $\mathbf{N}$.

Assume that $(\underline{q}_1)^n = \underline{C}_n$ is true for $n = k$, i.e., that $(\underline{q}_1)^k = \underline{q}_1 + \underline{q}_2 + \ldots + \underline{q}_k := \underline{C}_k$.

Then prove the "recursion clause", that $(\underline{q}_1)^{k+1} = \underline{C}_k \times \underline{q}_1 = (\underline{q}_1 + \underline{q}_2 + \ldots + \underline{q}_k) \times \underline{q}_1 = \underline{C}_{k+1}$.

So, $(\underline{q}_1)^{k+1} = (\underline{q}_1 \times \underline{q}_1) + (\underline{q}_2 \times \underline{q}_1) + \ldots + (\underline{q}_k \times \underline{q}_1) = (\underline{q}_1 + \underline{q}_2) + (\underline{q}_2 + \underline{q}_3) + \ldots + (\underline{q}_k + \underline{q}_{k+1})$

$$= (\underline{q}_1 + \underline{q}_1 + \ldots + \underline{q}_1) + \underline{q}_2 + \underline{q}_3 + \ldots + \underline{q}_k + \underline{q}_{k+1} = \underline{q}_1 + \underline{q}_2 + \underline{q}_3 + \ldots + \underline{q}_k + \underline{q}_{k+1} = \underline{C}_{k+1}.$$

Thus, by finite induction, the equation $(\underline{q}_1)^n = \underline{C}_n$ is true for all n in $\mathbf{N}$ for Multiplication Definition **2**.

Similarly for Definition **3**. Given $(\underline{q}_1)^n = \underline{C}_n$ for $n = k$, i.e. $(\underline{q}_1)^k = \underline{C}_{k-1} \times \underline{q}_1 = \underline{q}_1 + \underline{q}_2 + \ldots + \underline{q}_k := \underline{C}_k$,

prove that $(\underline{q}_1)^{k+1} = \underline{C}_k \times \underline{q}_1 = (\underline{q}_1 + \underline{q}_2 + \ldots + \underline{q}_{k-1} + \underline{q}_k) \times \underline{q}_1 = \underline{C}_{k+1}$.

So, $(\underline{q}_1)^{k+1} = (\underline{q}_1 \times \underline{q}_1) + (\underline{q}_2 \times \underline{q}_1) + \ldots + (\underline{q}_k \times \underline{q}_1) = (\underline{q}_1 + \underline{q}_2) + (\underline{q}_1 + \underline{q}_3) + \ldots + (\underline{q}_1 + \underline{q}_k) + (\underline{q}_1 + \underline{q}_{k+1})$

$$= (\underline{q}_1 + \underline{q}_1 + \ldots + \underline{q}_1) + \underline{q}_2 + \underline{q}_3 + \ldots + \underline{q}_k + \underline{q}_{k+1} = \underline{q}_1 + \underline{q}_2 + \underline{q}_3 + \ldots + \underline{q}_k + \underline{q}_{k+1} = \underline{C}_{k+1}.$$

Thus, by finite induction, the equation $(\underline{q}_1)^n = \underline{C}_n$ is true for all n in $\mathbf{N}$ for Multiplication Definition **3**.

Finally, for Definition **4**, Given $(\underline{q}_1)^n = \underline{C}_n$ for $n = k$, i.e. $(\underline{q}_1)^k = \underline{C}_{k-1} \times \underline{q}_1 = \underline{q}_1 + \underline{q}_2 + \ldots + \underline{q}_k := \underline{C}_k$,

prove that $(\underline{q}_1)^{k+1} = \underline{C}_k \times \underline{q}_1 = (\underline{q}_1 + \underline{q}_2 + \ldots + \underline{q}_k) \times \underline{q}_1 = \underline{C}_k$.

So, $(\underline{q}_1)^{k+1} = (\underline{q}_1 \times \underline{q}_1) + (\underline{q}_2 \times \underline{q}_1) + \ldots + (\underline{q}_k \times \underline{q}_1)$

$$= (\underline{q}_1 + \underline{q}_2 + \underline{q}_1) + (\underline{q}_2 + \underline{q}_3 + \underline{q}_1) + \ldots + (\underline{q}_k + \underline{q}_{k+1} + \underline{q}_1)$$

$$= (\underline{q}_1 + \underline{q}_1 + \ldots + \underline{q}_1) + \underline{q}_1 + \underline{q}_2 + \underline{q}_3 + \ldots + \underline{q}_k + \underline{q}_{k+1} = \underline{C}_{k+1}.$$

Thus, by finite induction, the equation $(\underline{q}_1)^n = \underline{C}_n$ is true for all n in $\mathbf{N}$ for Multiplication Definition **4**.